Optimisation models for planning in fuel management

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Ramya Rachmawati

Bachelor of Science (Gadjah Mada University)

Master of Science (Bogor Agricultural University)

Mathematical and Geospatial Sciences, School of Science

College of Science, Engineering and Health

RMIT University

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Declaration

I certify that except where due acknowledgement has been made, the work is that of the author alone; the work has not been submitted previously, in whole or in part, to qualify for any other academic award; the content of the thesis is the result of work which has been carried out since the official commencement date of the approved research program; any editorial work, paid or unpaid, carried out by a third party is acknowledged; and, ethics procedures and guidelines have been followed.

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Abstract

Uncontrolled wildfires can lead to loss of life and property and destruction of natural resources. Fuel management, or treatment planning by way of controlled burning or mechanical clearing, is an important tool used in many countries to reduce the risk of large wildfires. Management for fuel reduction should not be done in isolation of the ecological requirements of the ecosystem. Maintaining the ecological integrity of the landscape should also be considered. However, reducing fuel load in the landscape while maintaining ecological balance presents land managers with seemingly conflicting objectives. In this thesis, Mixed Integer Programming (MIP) models are developed to determine when and where fuel reduction activities should take place while maintaining vital ecological requirements of the landscape. The approaches are multi-period fuel treatment scheduling that tracks the age of each vegetation type and takes into account both the frequency of fire that it can tolerate and the frequency of fire necessary for fire-dependent species. The first model determines a long-term scheduling of the location for fuel treatment activities each year to minimise total fuel load over the planning horizon. The second model is formulated in such a way that it

breaks the connectivity of high-risk regions as a means to reduce fuel hazards in the landscape. The efficacy of the first two models was tested using randomised data from 711 public treatment units in the Barwon-Otway district of Victoria. The third model optimally schedules fuel treatment to fragment high-risk regions while ensuring sufficient habitat connectivity over time and space. This is critical for the conservation of fauna. This model is demonstrated in a series of computational experiments with a hypothetical landscape represented in grid cells. The formulation, however, is valid for real landscapes and provides the means to an integrated approach to ecosystem conservation and reducing the risk of large wildfires.

1 Introduction

1.1 Background

Fire is one of the fundamental components of many natural ecosystems, but if it is not well controlled, it can cause loss of human life and property, and destruction of natural resources (King et al., 2008). Across the globe, in the USA, Canada, Australia, and southern Europe, the frequency and intensity of destructive wildfires is projected to increase (Boer et al., 2009). The increasing temperatures and unpredictable weather conditions due to climate change suggest this trend will continue (Westerling et al., 2006; Wotton et al., 2003). The Victorian 'Black Saturday' bushfires in February 2009 which caused 173 fatalities and AUD$4 billion loss of assets, has become a warning that catastrophic wildfires threaten communities and the natural environment in fire-prone areas.

In an effort to lessen the risk posed by wildfires, fuel management programs have been extensively implemented in the USA (Ager et al., 2010; Collins et al., 2010) and Australia (McCaw, 2013; Boer et al., 2009). Fuel management is a method used to modify the structure and quantity of fuel. The management programs involve long-term planning of fuel reduction activities, such as prescribed burning and mechanical clearing (King et al., 2008; Finney, 2001; Loehle, 2004). Although fuel treatment alone cannot eliminate the potential of wildfires, this activity reduces the effort required for suppression and containment (Martell, 2015). The need for fuel treatment is likely to increase as fuel loads will always re-accumulate due to vegetation regrowth. Factors such as limited resources make it challenging to determine the optimal time and location of fuel treatment.

Models for solving landscape-level fuel treatment have been proposed in a number of studies. For example, Wei and Long (2014) proposed a single-period Mixed Integer Programming (MIP) model to disconnect high-risk patches by considering future fire spread speeds and durations. Minas et al. (2014) proposed a multi-period MIP model that breaks the connectivity of high fuel units in the landscape to prevent the fires spreading, however, this model is limited to a single vegetation type per treatment unit. In reality, a treatment unit may comprise a number of patches with different vegetation type and age. There is a lot of potential to develop previous fuel treatment models, for example, by proposing new multi-period models that take into account multi-vegetation types and ages within a treatment unit.

Management for fuel reduction becomes more complicated due to the recognition that this task should not be done in isolation of the ecological requirements of the ecosystem. Incorporating fuel treatment for fuel reduction burning and ecological requirements has received scant research attention (Penman et al., 2011). Therefore, it is important to develop models that bring together ideas from previous work and include ecological requirements.

1.2 Aims and objectives

The aim of this study is to develop models to determine the optimal time and locations to conduct fuel treatments while maintaining critical ecological requirements of the landscape. To achieve this aim, five detailed objectives are established:

Objective 1

To perform a literature review on the development of fuel management models over recent decades.

Objective 2

To develop a multi-period fuel reduction model applicable for a real landscape comprising multiple vegetation types.

Objective 3

To develop a multi-period fuel treatment model to disconnect high-risk areas in a real landscape. This is to extend the model to include a consideration of the spatial configuration of high fuel load areas. In particular, the aim is to develop a model that in each period minimises the measure of the connectivity of high-risk areas in a real landscape comprising multiple vegetation types.

Objective 4

To develop an integrated approach to disconnect high-risk areas and maintain habitat connectivity for animal species. In the process of fragmenting high fuel load areas, there is a possibility of leaving fauna without suitable habitat. The aim is to develop a model that ensures at each period there is a suitable spatial configuration of habitat for fauna.

Objective 5

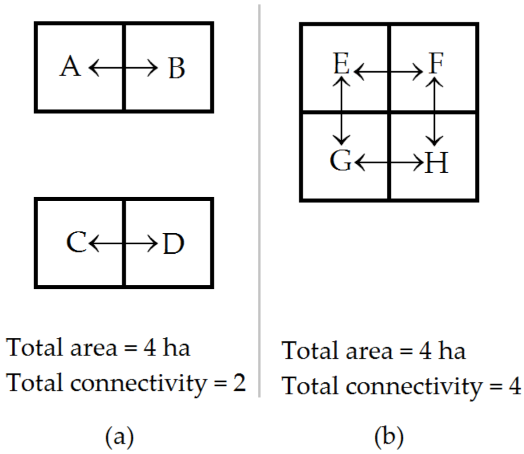
To identify possible future research directions that may improve the work in this study.

1.3 Scope of the thesis

1.3.1 The importance of fragmentation of high-risk areas

The approaches addressing objectives 3 and 4 of this thesis take into account the importance of the role of spatial arrangements in affecting the size of wildfires. We measure the size of potential wildfires by taking into account the connectivity of cells or treatment units in the landscape. Figure 1.1 illustrates this role with the example of two different four-hectare high-risk treatment units. In (a) there are two clusters of 2×1 high-risk treatment units, whereas in (b) there are 2×2 high-risk treatment units. Although both cases have the same area, the treatment units arrangement presented in (b) have more connectivity than (a). In (a), there are two connections, i.e. between A and B, C and D, whereas in (b), there are four connections: between E and F, E and G, G and H, F and H. Assuming that the fire will spread to the neighbouring high-risk areas, the cluster with more connectivity (b) will produce a larger fire. Therefore, to avoid large wildfires, we should fragment the high-risk areas.

Figure 1.1: An illustrative example of two different arrangements of four-hectare high-risk treatment units



1.3.2 The minimum and the maximum Tolerable Fire Interval (TFI)

The problem addressed by the models in this thesis is where and when to conduct fuel reduction while still considering the ecological requirements of the vegetation present. The ecological requirements can be described as the minimum and maximum Tolerable Fire Intervals (TFI). We assume that treating vegetation where its age is between these two intervals will maintain species diversity and hence support the ecosystem's health. The minimum TFI is defined as the minimum time required between two consecutive fire events at a location and is normally based on the time to reach maturity of the sensitive species in the vegetation class, while the maximum TFI refers to the maximum time needed between fire events at a location that considers the fire interval required for fire-adapted species rejuvenation (Cheal, 2010). In this thesis, we use vegetation age to represent these TFI values. A treatment unit should not be treated if the age of vegetation growing in that location is under minimum TFI. In contrast, treatment units with vegetation over the maximum TFI must be treated.

1.4 Structure of the thesis

This thesis is structured as follows:

In Chapter 2, we address Objective 1 of this thesis. The complexity of fuel management planning and the efforts necessary to deal with this is evaluated with some examples drawn from published papers.

In Chapter 3, we address Objective 2 of this thesis. A MIP model is proposed to minimise fuel load over the planning horizon. The model takes into account multiple vegetation types in the landscape. The proposed model is run using an exact MIP (using a commercial MIP solver) and two heuristic approaches that break down the problem into multiple single-period sub problems. The model is tested using randomised data from 711 treatment units in the Barwon-Otway district of Victoria, Australia comprising different shapes and sizes of treatment units.

In Chapter 4, we address Objective 3 of this thesis. A MIP approach is formulated to reduce the spatial connectivity of fuel hazards while still considering the ecological fire requirements of the ecosystem. This approach is formulated in such a way that it breaks the connectivity of high-risk regions as a means to reduce fuel hazards in the landscape. Previous work is extended by modelling multiple vegetation types implemented within a polygon-based network. Thereby a more realistic representation of the landscape is achieved. The approach is then applied using randomised landscape data from the Barwon-Otway district in south-west Victoria, Australia for fuel treatment planning.

In Chapter 5, we address Objective 4 of this thesis. A MIP approach is proposed that can schedule fuel treatment to reduce fuel hazards by fragmenting high-risk regions, while maintaining the habitat connectivity and availability. The objective is to schedule fuel treatment to break the connectivity of high-risk areas while minimising the negative impact of fuel treatment on the ecosystem. Importantly, our approach ensures that at the time an area is treated a suitable neighbouring habitat is available to allow fauna to relocate. Furthermore, the model sets a minimum acceptable target for habitat connectivity at any time to conserve fauna. This complex decision making is then demonstrated and analysed in a series of computational experiments with a hypothetical landscape of grid cells. However, the formulation is also valid for real landscapes.

Finally, Chapter 6 presents the conclusion of this thesis with research findings and recommendations for future directions.

1.5 List of publications

The following are publications based on the contents of this thesis:

Rachmawati, R., Ozlen, M., Reinke, K. J., & Hearne, J. W. (2015). A model for solving the prescribed burn planning problem. *SpringerPlus*, 4(1), 1-21.

Rachmawati, R., Ozlen, M., Reinke, K. J., & Hearne, J. W. (2015). An optimisation approach for fuel treatment planning to break the connectivity of high-risk regions. *Forest Ecology and Management*. Pending for publication with minor revisions. The preprint is available at http://arxiv.org/abs/1512.08453

Rachmawati, R., Ozlen, M., Hearne, J. W., & Reinke, K. J. (2016). Fuel treatment planning maintaining habitat availability and connectivity for endangered species conservation. The preprint is available at http://arxiv.org/abs/1602.00348

Other publication during candidature:

Rachmawati, R., Ozlen, M., Hearne, J. W., & Kuleshov, Y. (2014). Using improved climate forecasting in cash crop planning. *SpringerPlus*, 3(1), 1-7.

2 Literature review

This section provides a literature review on the development of fuel management over the last few decades. More detailed literature reviews associated with the research objectives of this thesis are presented in the introductory sections of chapters 3 to 5.

2.1 Introduction

Fuel management, or treatment planning by way of prescribed burning or mechanical clearing, has been widely used as an effective way to reduce fuel accumulation in a landscape (Agee and Skinner, 2005). While the importance of fuel treatment has been widely recognised, how to decide when and where to conduct this management program optimally is not straightforward (Rönnqvist et al., 2015). This complex activity requires management strategies that link both spatial and temporal aspects (Bettinger, 2010; Belval et al., 2014). Factors that should be considered include limited resources, land ownership, landscape heterogeneity, varying fuel accumulation rates, locations of values-at-risk, probability of ignitions, availability of habitat for endangered species, and minimum and maximum tolerable fire intervals. Because of this complexity of the fuel treatment scheduling, most models or tools that have been developed today take into account only a limited number of these factors (Chung, 2015).

Integrated modelling approaches that link both spatial and temporal changes have not yet been widely developed and implemented (Chung, 2015). For instance, the model developed by Reinhardt et al. (2003) only consider the vegetation growth over time, without incorporating the importance of the location to avoid fire spreading between stands in the landscape. Omitting this factor in the model makes the effects of fuel treatment at the landscape level difficult to evaluate. Contrary to the model proposed by Reinhardt et al. (2003), there are many other models of computer simulation, such as NEXUS (developed by Scott (1999)), FARSITE (developed by Finney (2004)), FlamMap (developed by Finney (2006)) and FIREHARM (developed by Keane et al. (2010)), that solely take into account the spread of fire across a landscape, without incorporating the temporal aspect of fuel structure dynamics in the landscape. Again omitting this factor makes a long-term plan for fuel treatment unaddressed. Since many of the models are complementary and can be used in combination with one another, fire and land manager who needs to decide the time and location for conducting fuel treatment to mitigate future risk of wildfires need to run several models to achieve the most effective fuel treatment planning (Chung, 2015).

Operations Research (OR) is a discipline that uses analytic methods to analyse complex problems to help make better decisions (Altay and Green, 2006). It provides great value in assisting fire and land managers in evaluating alternatives and making decisions for fuel reduction over the landscape. In this chapter, we review OR approaches for fuel treatment planning over the last few decades.

We identify their assumptions and simplifications to handle this complex task for fuel treatment planning and their contributions to the body of knowledge. We present previous modelling efforts that have been undertaken in fuel treatment planning. We particularly focus on the development of operations research approaches drawn from fire and forest management literature. The approaches can broadly be categorised as simulation/heuristic optimisation, multi-objective optimisation, dynamic programming and mixed integer programming.

2.2 Simulation/heuristic optimisation

The simulation approach is used to model real-world processes that evolve stochastically over time. Simulations can be applied to determine possible alternatives when the decision makers face a complex and uncertain decision environment. Decision makers can also use heuristic optimisation to find good solutions of a specified system iteratively. We present some examples of simulation/heuristic approaches in fuel treatment planning as follows.

Jones et al. (1999) integrated the simulation and optimisation model to measure the fuel treatment effects on reducing future wildfires extent and severity while maximising net revenue. They developed a simulation model SIMPPLLE (simulating vegetative patterns and processes at landscape scales) to assess wildfire risks if there is no fuel treatment, but with fire suppression. The results from the SIMPPLLE model become the input into the optimisation model MAGIS (multi-resource analysis and geographic information system) to reduce fire hazards while achieving other management objectives. Although their approach does not take into account the effort to stop the spread of fire, their work provides useful insights into better understanding, managing and monitoring fire-prone forested landscapes. A study that takes into account the effort to stop the fire spread has been conducted by Finney et al. (2008). They proposed a system that incorporates both spatial and temporal factors to fuel treatment planning at a landscape level. They utilised the previous work by Finney (2007) for each single planning period to locate the fuel treatment to efficiently disrupts the spread of fire. The results of this model become an input for the Forest Vegetation Simulator (FVS) in updating the changes of vegetation and fuels. The output of this FVS becomes the input of the first model. This process continues, forming a 'long-term' fuel treatment plan. Although this approach can schedule multiple-period fuel treatment planning, the location for treatments in each period do not take into account the continuing effects in the following periods (Chung et al., 2013). Addressing this shortcoming, Chung et al. (2013) developed a decision support system, OptFuels, that incorporated the fire and vegetation models into an optimisation system to evaluate spatio-temporal effects of fuel treatments. Their simulation system is similar to the system used by Finney et al. (2008). However, to solve the large combinatorial problem, OptFuels apply a simulated annealing heuristics method and take into account continuing spatial and temporal effects of treatments. They also take into account forest dynamics over time, value at risk, fire spread and behaviour, and specific method for fuel treatments. While their approach does not guarantee the optimal solution, it improves the quality of the solutions over the search process. However, the ignition is still deterministic, and the impacts of future wildfires are not considered.

As an alternative, some researchers have used a fire-spread simulator to schedule fuel treatments. For example, Kim and Bettinger (2008) used a fire-spread simulator to evaluate whether there is an impact if a fuel treatment is scheduled on a broad-scale landscape. They tested their approach in four scenarios, namely dispersed, clumped, random and regular on a real landscape. They concluded that based on the operational point of view, the clumped pattern may be the most effective and efficient scenario, and the random pattern provides no effect on reducing the simulated human-caused wildfire risk. Later, Kim et al. (2009) utilised a heuristic optimisation method in landscape-level timber management. Using the same four scenarios as in the study by Kim and Bettinger (2008), they concluded that despite the spatial arrangement of harvesting units, their approach is not effective to achieve timber management objectives while trying to mitigate wildfire behaviour in a heterogeneous landscape. González-Olabarria and Pukkala (2011) proposed an iterative optimisation approach utilising a fire-spread simulator and a simulated annealing algorithm to schedule fuel treatment with the idea to stabilise fire risk and net revenue. Their approach is applied to homogeneous hexagon cells with different land use in North-East Spain. In this approach, fuel treatment method is done by timber harvesting only and exclude other methods such as prescribed burning and mechanical thinning.

Garcia-Gonzalo et al. (2014) included the thinning treatments in their proposed simulation-optimisation approach in reducing the fire risk while optimising stand management. They confirm that by using their method, the profitability increases and the expected damage decreases.

2.3 Multi-objective optimisation

Fuel management involves various priorities that may conflict, for instance fuel treatment can reduce fire hazards but may cause negative impact on habitat availability for some species (Martell, 2007). Multi-objective optimisation or multi-criteria decision making works well to handle multiple conflicting objectives and some researchers have applied the multi-objective optimisation approach to fuel management. For example, Lehmkuhl et al. (2007) used FuelSolve (an optimisation model for fire spread) and an evolutionary algorithm to simultaneously minimise the potential risk of fire while maximising the availability of habitat for endangered species. Kennedy et al. (2008) also used FuelSolve to assess the trade-offs between these objectives: Protect habitat for endangered species, preserve old growth forest reserves, and minimise the total treated area. Calkin et al. (2005) used a goal programming method to evaluate the trade-offs between reduction of fire risk and maintaining habitat in silvicultural treatment planning. Their results suggest that it is possible to achieve both goals if the late-seral forest for habitat of animal species is 45% or less.

2.4 Dynamic programming

Dynamic programming method is used for solving complex problems by breaking up a large problem into smaller, tractable subproblems (Winston and Goldberg, 2004). Some studies have used dynamic programming in fuel treatment planning. Ferreira et al. (2014) used stochastic dynamic programming (SDP) to determine the optimal strategy to maximise expected net revenue at stand management level by integrating models for vegetation growth and wildfire events. Konoshima et al. (2008) also proposed an SDP model that can maximise future timber production by considering the future fire events and spreads into fuel treatment planning. In follow up paper, Konoshima et al. (2010) extended their previous model by including factors such as weather condition and topography, and then conducted the model demonstrations with a hypothetical landscape comprising homogeneous hexagonal units. They found out that the spatial arrangement of management units led to differing management strategies.

2.5 Mixed Integer Programming

Optimisation approaches can be used to optimally determine time and location for strategic (long-term) fuel treatment planning across landscapes (Nguyen, 2015). Mixed Integer Programming (MIP) methods deal with the optimisation of some explicit and measurable objective (Williams, 2009). This objective is defined as a mathematical function of the decision variables in the form of an 'objective function' and is optimised, can be maximised or minimised, subject to a series of related constraints (Hillier and Lieberman, 2001). Some of the decision variables are integers and other variables are allowed to be non-integers. MIP can be effective to model problems that involve: 'yes or no' decisions or logical connections such as 'if-then' constraints (Wolsey, 1998).

There are previous efforts for solving fuel treatment planning using MIP approaches. For example, Hof et al. (2000) and more recently Hof et al. (2002) formulated MIP models for fuel treatment planning to delay the fire spread from its deterministic ignition point to one or more protecting locations. Wei et al. (2008) applied a piece-wise linear approximation of the occurrence and spread of fire in a MIP model to decide on optimal fuel treatment location to reduce expected loss incurred on a landscape. Wei (2012) later proposed a MIP method to locate fuel reduction treatments to set up potential control locations for future fires. In their recent paper, Wei et al. (2014) formulated a single-period spatial optimisation model to fragment high-risk patches by taking into account the future ignition probability of each fire to minimise expected loss. Acuna et al. (2010) incorporated stochasticity of fire into their MIP model in dealing with fire and forest management planning to maximise revenue. Minas et al. (2013) developed a MIP model that integrates fuel treatment and suppression to maximise their simultaneous effects on a single-period wildfire preparedness planning. More recently, Minas et al. (2014) proposed a multi-period MIP model to optimally locate fuel treatment to break the connectivity of high-risk treatment units applied in a landscape comprising rectangular grid cells with a single vegetation type within each cell. The model tracked the age of vegetation for both treated and untreated cells over the landscape across the planning horizon. Their model was then extended by incorporating some ecological requirements such as the proportion of vegetation age in the landscape.

Optimisation approaches have been used to optimally select good solutions for fuel treatment planning. However, this process remains challenging (Martell, 2007) because it requires a lot of computing power. The improvements of computation power and optimisation algorithms have attracted researchers to use exact optimisation approaches for solving more and more complex fuel management problems. However, particularly when facing a difficult problem, the decision maker can combine optimisation with heuristics to find near-optimal solutions (Borges et al., 2002).

2.6 Chapter summary

In this chapter, we have presented previous modelling efforts that have been undertaken in scheduling fuel management programs. Some studies have developed fuel treatment models for a single vegetation type (Minas et al., 2013, 2014; Hof et al., 2002) and single-period fuel treatment models (Wei et al., 2014; Finney et al., 2008). For real landscapes, it is common that there are different vegetation types and ages within a treatment unit. Consequently, it is important to incorporate multiple vegetation types in fuel treatment planning. Due to the transience of fuel load in the landscape for both treated or untreated areas, it is also important to model multi-period planning strategies. Furthermore, with the exception of the work by Calkin et al. (2005); Lehmkuhl et al. (2007); Kennedy et al. (2008), most of the reviewed models do not take into account the habitat availability for fauna when conducting fuel treatment.

3 A model for solving the prescribed burn planning problem

3.1 Introduction

Fuel management is a complex activity that involves both spatial and temporal decisions (Belval et al., 2014) and handling multiple fuel and ecological objectives. The development of decision support tools for fuel management programs is an ongoing and active research area (Martell, 2011). As an example, Wei et al. (2008) formulated an integer programming approach to reduce expected loss incurred on a landscape. Wei (2012) later proposed a Mixed Integer Programming (MIP) method to locate fuel reduction treatments to set up potential control locations for future fires. Minas et al. (2014) developed a model that deals with fuel treatment scheduling to break the connectivity of high risk treatment units applied in a landscape. However, a limitation of the models proposed, such as that by Minas et al. (2014), is that it only handles a single vegetation type, and fuel accumulation is treated as a linear function of time. In reality, the fire landscape is made up of multiple vegetation types, of mixed ages, with fuel accumulation taking on non-linear functions depending on vegetation type. The model presented in this chapter addresses these limitations by formulating a model within a landscape that consists of multiple vegetation types of mixed ages, with differing non-linear fuel accumulation functions.

A recent review paper written by Chung (2015) highlighted the complexity of fuel treatments and examined previous fuel treatment optimisation studies to deal with it. Of note is that few studies incorporate the spatial and temporal dimensions of the problem. Perhaps more importantly is the conclusion that "most existing optimisation models suffer from problem complexity and a computationally intensive process ... making them almost impractical for field applications" (Chung 2015, p.50). There is a clear need to understand the fitness for purpose of our models and to move beyond proof of concept applications. Do the trade-offs warrant obtaining the perfect solution? Can we obtain a near-optimal solution with heuristic approaches? To answer these questions, we illustrate both exact and approximate methods with a series of computational experiments with a case study of the Barwon-Otway district of Victoria, Australia. We develop a Mixed Integer Programming (MIP) model for prescribed burn planning. The objective function of the model is to reduce fuel load accumulation in a landscape of multi-age, multiple vegetation types, with differing non-linear fuel accumulation functions.

The complex multi-period model proposed in this chapter can be solved exactly using an MIP solver or can be decomposed into single-period sub problems.

The single-period sub problems are solved exactly using a solver and approximately using a greedy heuristic. With the exact MIP approach, an optimal solution can be achieved. However, the computational effort is costly. We introduce the two heuristics because the problem is NP-hard. With the single-period heuristic approaches, less computational effort is needed, but the solution may not be optimal as the exact MIP approach. The three approaches for solving the model are compared in terms of model applicability, computational time and the objective values.

3.2 Model formulation

In this chapter, candidate locations for fuel reduction burns are represented by 'treatment units'. A treatment unit is defined as any area of land considered suitable for a planned burn treatment. Private land and water bodies, such as rivers and lakes, are considered non-treatable areas and are excluded from the model. Within the data set, a treatment unit is represented as a spatial feature or polygon and contains additional attributes relating to the land ownership, vegetation types, vegetation ages and geometric properties, such as size, that exist within that treatment unit. The treatable area within the treatment unit is defined as the areas that have non-zero fuel loads.

The prescribed burning planning problem in this chapter is NP-hard. The Knapsack Problem (KP), a well-known NP-hard problem (Garey and Johnson, 1979), can be transformed to the one-year planning horizon Prescribed Burn Planning (PBP) problem in polynomial number of steps. The objective function of the KP is to maximise total profit, i e. given a set of items, each with a weight and profit, determine the items to include so that the total weight is less than or equal to a given capacity limit. In order to transform KP into PBP, the capacity limit of the regular KP is changed to the burn limit. The items are transformed to the treatment units; the weight of the items is changed to the areas, and the profits become the fuel loads. The minimum and the maximum TFI of the problem are set to infinity.

We consider the landscape divided into treatment units. It is assumed that all the vegetation of each kind is of the same age within each treatment unit. With the decision to determine when and where to treat every year to minimise total fuel load of certain regions, the following mixed integer programming model is formulated.

Sets:

V_i is the set of vegetation types growing in treatment unit i

T is the planning horizon

C is the set of treatment units of which total fuel load is to be minimised

Indices:

i = treatment unit

j= vegetation type

k = vegetation age

t = period, $t = 0, 1, 2, \ldots$

Parameters:

w_i = relative importance (weight) of treatment unit i

$m_{i,j}$ = the age of vegetation type j in treatment unit i at the beginning of the time-period

$A_{i,j}$ = area (in hectares) of treatment unit i with vegetation type j

R = the total treatable area in a landscape

ρ = treatment level (in percentage), i.e. the maximum proportion of the total treatable area in a landscape selected for treatment

c_i= area of treatment unit i (where $c_i = \sum_j A_{i,j}$)

$L_{j,k}$ = fuel load (ton/hectare) of vegetation j, at age k

$maxTFI_j$= maximum TFI of vegetation type j

$minTFI_j$ = minimum TFI of vegetation type j

Decision variables:

$$x_{i,t} = \begin{cases} 1 & \text{if treatment unit } i \text{ is treated in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$y_{i,j,k,t} = \begin{cases} 1 & \text{if in treatment unit } i\text{, there is vegetation type } j\text{, at age } k\text{, in time } t \\ 0 & \text{otherwise} \end{cases}$$

minimise total weighted fuel load

$$z = \sum_{t=1}^{T} \sum_{k} \sum_{j \in V_i} \sum_{i \in C} w_i L_{j,k} A_{i,j} y_{i,j,k,t} \tag{3.1}$$

subject to

$$y_{i,j,k,0} = 1,\ \forall i, j \in V_i,\ k = m_{i,j} \tag{3.2}$$

$$y_{i,j,k+1,t+1} \geq y_{i,j,k,t} - x_{i,t},\ \forall i, j \in V_i,\ k\ = 1, 2, \ldots, maxTFI_j - 1, \forall t \tag{3.3}$$

$$y_{i,j,k,t} \leq x_{i,t},\ \forall i, j \in V_i, \forall t, \text{ for } k = maxTFI_j \tag{3.4}$$

$$y_{i,j,1,t+1} \geq x_{i,t},\ \forall i, j \in V_i, \forall t \tag{3.5}$$

$$\sum_k y_{i,j,k,t} \leq 1,\ \forall i, j \in V_i, \forall t \tag{3.6}$$

$$\sum_{j \in V_i} \sum_{k < minTFI_j} y_{i,j,k,t} - \mid V_i \mid \sum_{j \in V_i} \sum_{k = maxTFI_j} y_{i,j,k,t} \leq \mid V_i \mid (1 - x_{i,t}),\ \forall i, j \in V_i, \forall t \tag{3.7}$$

$$\sum_i c_i x_{i,t} \leq \rho R,\ \forall t \tag{3.8}$$

$$y_{i,j,k,t} \in \{0, 1\} \tag{3.9}$$

$$x_{i,t} \in \{0, 1\} \tag{3.10}$$

The objective function (3.1) minimises the weighted total fuel load of all vegetation at all regions throughout a planning horizon.

Constraint (3.2) sets the initial conditions. Based on our observation of some raw data we felt it was necessary to include the possibility that the different vegetation types might differ in their ages. However, we assume that the all vegetation of a given type within a treatment unit will be of the same age. Constraint (3.3) indicates that when $x_{i,t} = 0$, which means fuel treatment is not conducted, the vegetation in that area will continue growing until the following period, and the age will be incremented by one.

Constraint (3.4) ensures that vegetation will be treated once it has reached maximum TFI. The vegetation with age 1 in the next period comes from the areas that are treated in the current period, as denoted in constraint (3.5). Constraint (3.6) ensures that in each time-period all vegetation of a specific type in each treatment unit will be of the same age. In reality, the same vegetation type within a treatment unit may have different ages resulting from wildfires that have burnt a treatment unit partially. However, we assume that there is a representative dominant age for each vegetation type in a treatment unit. Considering the possibility of multiple ages of the same vegetation type would be computationally prohibitive. Constraint (3.7) enforces that the vegetation under minimum TFI cannot be treated unless there is another vegetation type in the same treatment unit which is over the maximum TFI to avoid a deadlock. However, if required, this constraint can be changed to the other way, i.e. treatment units containing young treatment units cannot be treated. Here $\mid V_i \mid$

represents the number of different vegetation types in treatment unit i.

Constraint (3.8) specifies that the total area selected for fuel treatment each year is not more than the annual area allotted (target) for fuel treatment (in hectares). Here, the target is obtained by multiplying the treatment level and the total treatable area in a landscape. Constraint (3.9) and (3.10) ensure that the decision variables $y_{i,j,k,t}$ and $x_{i,t}$ take binary values.

The model is capable of handling multiple vegetation types and ages. Each vegetation type has different minimum and maximum TFI, and at any period each vegetation type may have a different age even within a single treatment unit. The fuel curve representing each age of certain vegetation can also be a nonlinear function.

3.3 Solution Approaches

3.3.1 An exact Mixed Integer Programming (MIP) approach

The multi-period model discussed in Section 3.2 can be solved exactly using an MIP solver. In this subsection, the model improvement is presented to enhance the solution time.

The solution time of a mixed integer programming problem can generally be improved by reducing the number of variables, or restricting the values that they can take. Age index k should be based on the set of possible ages that vegetation type j can take in treatment unit i at time t. The maximum possible periods between two consecutive treatments for any treatment unit can be derived by finding the minimum of the maximum TFI values of all vegetation types available within that unit. This sets an upper limit on the values k can take within that treatment unit.

We can also tighten the mixed integer programming formulation by introducing valid inequalities on the frequency of treatment event in each unit as follows

$a_{i,j}$ = initial age of vegetation type j at treatment unit i

$q = \min(maxTFI_j - a_{i,j})$

$$\sum_{t=0}^{q-1} x_{i,t} \geq 1, \ \forall i \tag{3.11}$$

$p = \min(maxTFI_j)$

$$\sum_{t}^{t+p-1} x_{i,t} \geq 1, \ \forall i \ for \ t = 0, 1, ..., T - p \tag{3.12}$$

$$x_{i,t} = 0, \ \forall i, \forall t \text{ such that } t < \min(\min(minTFI_j - a_{i,j}), \min(maxTFI_j - a_{i,j})), \ j \in V_i \tag{3.13}$$

Constraint (3.11) ensures that a treatment unit will be treated when the most critical vegetation type (i.e., the vegetation type which sets the minimum of the maximum TFI value among all vegetation types available within a treatment unit) reaches its maximum TFI. In other words, we have to treat the treatment unit at some time in the first q periods of the planning horizon. Constraint (3.12) generalises this idea to the rest of the planning horizon by setting a frequency to treat. It ensures that treatment unit i must be treated at least once every p years. It is assumed that each treatment unit has a critical vegetation type (i.e. the vegetation in the treatment unit which has the least maximum TFI) that determines the treatment cycle. However, constraint (3.12) can only help to speed up the computation time when the planning horizon is longer than the burning frequency in the treatment units. Constraint (3.13) reduces the number of binary variables by setting the burn variables to 0 for burns that are not allowed based on the TFI values. We considered improving the solution time by treating variable $y_{i,j,k,t}$ as a continuous variable instead of a binary variable. In other words, we replace constraint (3.9) with constraint (3.14) as follows.

$$0 \leq y_{i,j,k,t} \leq 1 \tag{3.14}$$

3.3.2 Single-period heuristic approach

In this subsection, two single-period heuristic approaches: an exact method for the single-period problem and an approximate method for the single-period problem are presented. These approaches, which are a single period 0/1 knapsack problem and a basic 'greedy' algorithm, are conducted as follows.

Consider I is the set of all treatment units in the landscape. The landscape is grouped into three disjoint sets: I_{old}, I_{middle} and I_{young}. The first set, I_{old}, is the set of treatment units where at least one of the vegetation ages are over the maximum Tolerable Fire Interval (TFI). The second set, I_{middle}, is the set of treatment units where the vegetation ages are between the minimum and the maximum TFI, and nothing is over maximum TFI. The third set, I_{young}, is the set where all vegetation ages under maximum TFI and at least one vegetation under the minimum TFI. Here, $I = I_{old} \cup I_{middle} \cup I_{young}$. Using these parameters,

A_i is area of treatment unit i

R is the total treatable area of the landscape

ρ is treatment level (in percentage),

then the value of $r = \sum_{i \in I_{old}} A_i$ can be determined. There are two cases that may arise when comparing the values of r and ρR.

3.3.2.1 Case 1: $r \geq \rho R$

If $r \geq \rho R$, then $x_i = 0$, for $i \in I_{middle} \cup I_{young}$. Either of these two approaches may be applied:

Using an exact method for the single period problem

The next step is to run the following model, maximise (3.15) subject to (3.16), with i is defined only for I_{old}. Here, ρ_{new} is the new treatment level (in percentage), where $\rho_{new} = \rho$.

maximise total fuel load:

$$z = \sum_i L_i x_i \tag{3.15}$$

subject to

$$\sum_i A_i x_i \leq \rho_{new} R, \tag{3.16}$$

where L_i is the total fuel load of treatment unit i, and x_i is a binary variable, that is

$$x_i = \begin{cases} 1 & \text{if treatment unit } i \text{ is treated} \\ 0 & \text{otherwise} \end{cases}$$

The objective function (3.15) is to maximise the total fuel load of all treatment units to be treated, subject to the single constraint (3.16).. This constraint limits the area that can be treated per year. The model will choose the treatment units containing the highest fuel load to be burned each year. Note that the objective function (3.15) is different from the original objective function (3.1). The objective of the original problem is to minimise the total fuel load that remain in the landscape. Conversely, the objective of the single-period problem is to maximise the total fuel load that can be taken from the landscape.

Using an approximate method for the single-period problem

The treatment units are sorted based on the highest fuel load per area of treatment unit in the landscape, hence determining the rank or priority to burn. The treatment units then are selected by this rank until the burn limit requirement, ρR, is met.

"Using an exact method for the single period problem" provides an exact solution using Integer Programming and "Using an approximate method for the single-period problem" provides an approximate solution based on the exact solution of the continuous knapsack problem.

3.3.2.2 Case 2: $0 \leq r < \rho R$

If $0 \leq r < \rho R$, then $x_i = 1$ for $i \in I_{old}$. Either of these two approaches may be applied:

Using an exact method for the single period problem

The next step is to maximise (3.15) subject to (3.16) with i is defined only for I_{middle}. Here, $\rho_{new} = \rho - \frac{r}{R}$.

Using an approximate method for the single period problem

The same process of ranking and selecting as with the Case 1 in "Using an approximate method for the single-period problem" is undertaken until the burn limit requirement, $\rho R - r$, is met.

The approximate method can fail if we cannot use the capacity fully. The performance should get better if we have many small treatment units that we can burn to use the capacity (almost) fully.

3.4 Model demonstration

Consider a landscape divided into 40 treatment units. The area of each treatment unit, vegetation type and age are described in Table 3.1. The data regarding the minimum and the maximum TFI and the fuel type of each Ecological Vegetation Class (EVC), can be seen in column two to five in Table 3.2. Figure 3.1 represents the fuel curve for each age of the certain vegetation type. Based on this data, some computational experiments were conducted to demonstrate three approaches: the exact MIP, the exact single-period and the approximate single-period problem. For the three approaches, we ran five and ten percent treatment levels, with and without TFI requirements. Figure 3.2 represents the fuel treatment schedule for the five-year planning horizon with TFI requirements. The total fuel load resulting from the experiments for the five-year planning horizon is represented in Figure 3.3.

From these figures, it is clear that the ten percent treatment level results in less total fuel load than that of the five percent treatment level. For this small landscape with the five-year planning horizon and with TFI, the three approaches show no substantial differences, which is most likely due to the relatively small feasible region. Without TFI requirements, the feasible region will be larger than if TFI is included. This larger feasible region makes the exact MIP approach superior to the other two approaches.

We also ran experiments for ten and 15-year planning horizon with the three approaches, with and without TFI requirements. Table 3.3 represents the solution times and objective values for these experiments. The solution time rises as the length of the planning horizon expands. The approximate approach for the single-period problem has the lowest solution time of all, but the solution quality is also less than the other two approaches. In this small landscape, the exact approach for the single-period problem does not always outperform the approximate approach for the single-period problem, because of factors such as randomness and size of treatment units.

The results for the five and ten percent treatment levels with ten-year planning horizon with TFI are described in Figure 3.4. This figure shows that for each treatment level, the result of the exact MIP approach and the exact single-period problem shows no substantial difference.

Overall, in this small landscape, the result obtained by the exact method for the single-period problem is as good as that of the exact MIP approach. In

Table 3.1: 40 treatment units data containing vegetation type, extent and age

Treatment unit ID	EVC code	area (ha)	age (years)
96	20	6.07	14
96	164	5.52	14
96	21	11.02	14
96	22	1.02	14
96	55	0.72	14
115	71	26.23	35
127	161	0.51	2
127	3	24.64	2
139	233	1.53	3
139	45	21.79	3
139	30	2.06	3
169	8	6.92	5
169	16	21.78	5
180	45	25.79	8
192	45	24.63	81
236	16	5.96	5
236	48	20.52	5
277	16	26.95	52
298	48	12.92	12
298	161	5.28	37
298	6	6.94	37
306	48	26.02	37
306	161	1.69	37
310	48	22.49	37
310	161	1.70	37
346	48	8.81	38
346	16	13.01	38
346	198	1.36	38

Treatment unit ID	EVC code	area (ha)	age (years)
351	16	26.61	37
351	48	0.76	37
376	48	25.18	37
384	175	23.86	2
403	48	24.01	37
477	16	24.76	6
602	16	20.27	6
602	48	2.59	6
602	23	1.84	6
634	16	27.27	36
796	8	18.33	10
796	48	10.25	10
813	16	4.03	37
813	45	14.39	37
813	21	9.30	37
831	48	3.47	6
831	16	24.16	6
831	198	1.26	6
833	48	2.89	2
833	1	2.56	2
833	6	1.58	2
833	161	18.32	2
987	16	7.29	1
987	8	3.04	1
987	45	16.09	1
1033	45	25.74	81
1035	3	2.03	37
1035	161	17.48	37

Treatment unit ID	EVC code	area (ha)	age (years)
1035	163	5.62	37
1049	16	7.97	16
1049	48	18.69	37
1081	48	14.97	25
1081	16	6.82	25
1081	178	2.22	55
1093	48	13.15	37
1093	161	12.81	37
1093	163	1.15	37
1107	22	23.59	2
1107	47	1.48	2
1121	20	26.43	6
1125	20	14.15	5
1125	22	13.20	5
1130	20	27.54	14
1134	16	13.79	3
1134	20	1.60	3
1134	22	7.72	3
1151	20	25.69	15
1152	20	18.74	4
1152	47	8.33	4
1171	851	3.39	3
1171	20	4.91	3
1171	22	19.55	3
1179	20	25.65	27
1181	16	19.08	8
1181	20	7.29	8

Figure 3.1: Fuel load accumulation curves over time for different fuel types listed for the Barwon-Otway region

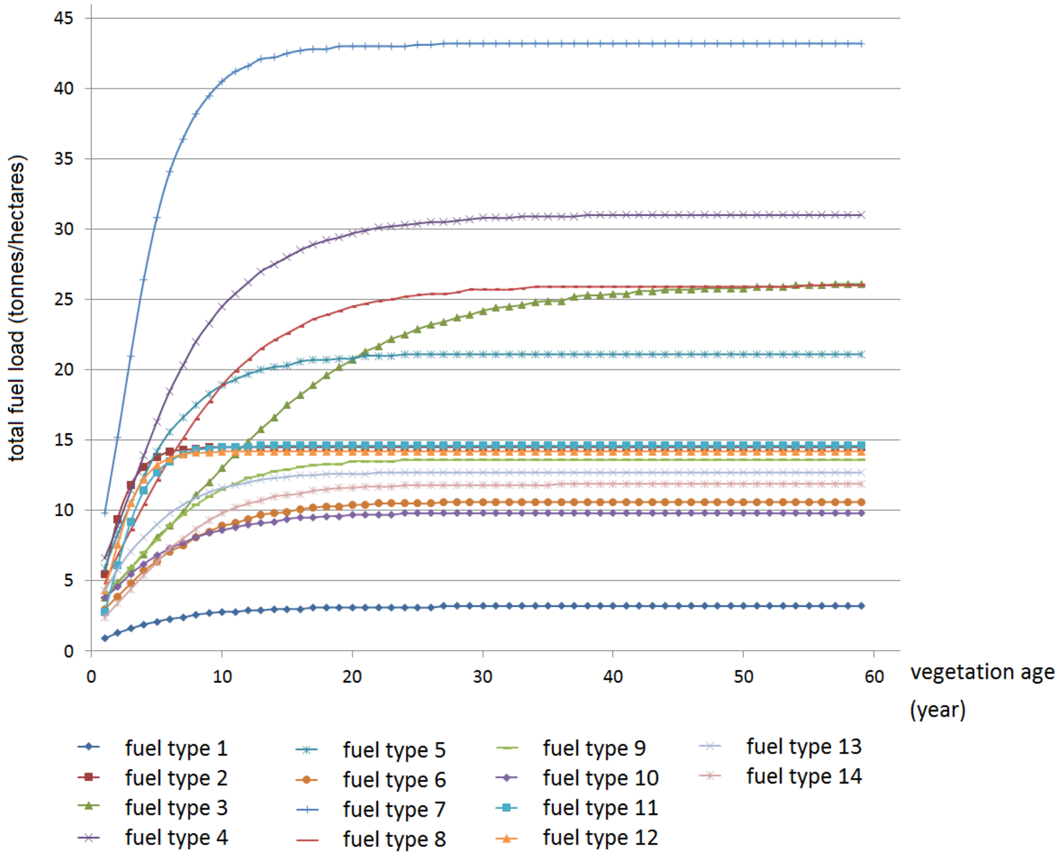


Figure 3.2: Fuel treatment outcomes, for a five percent treatment level (40 treatment units)

	year 1	year 2	year 3	year 4	year 5
Exact MIP approach					
treated treatment unit ID	351 and 403	1151 and 1179	96 and 634	1093 and 346	1035 and 1049
reduced fuel load (tonnes)	1439.02	1501.64	1403.64	1143.30	1020.06
Exact method for the single-period problem					
treated treatment unit ID	1151 and 1179	96 and 634	346 and 351	1093 and 403	1035 and 1049
reduced fuel load (tonnes)	1488.80	1394.37	1484.75	1109.25	1020.06
Approximate method for the single-period problem					
treated treatment unit ID	277 and 346	96 and 634	351 and 403	1093 and 1081	1035 and 1179
reduced fuel load (tonnes)	1473.887	1394.37	1451.59	1106.95	1101.04

Section 3.5, the three approaches are applied in a larger landscape.

3.5 An Australian case study

An Australian case study is presented to demonstrate the model. The study location is situated in the Barwon-Otway district of Victoria, Australia, and covers approximately 1,150,000 hectares (Figure 3.6a). Data used in this case study considers land ownership, vegetation type and age in each treatment unit, minimum and maximum TFI, and fuel load for the specific age of vegetation. In this case study, we categorise the treatment units according to land ownership (i.e. public or private). It is assumed that treatments can only occur on public land, so the candidate locations for prescribed burn planning are represented in these treatment units only. A total of 711 of treatment units exist over 73,535 hectares. Figure 3.6b shows the public land treatment units.

Table 3.2: Ecological Vegetation Class (EVC) and associated fuel types

EVC name	EVC code	min TFI	max TFI	fuel type	area (hectare)	area (per-cent-age)	initial fuel load (ton)
Creekline Grassy Woodland	68	20	150	7	6.14	0.008	65.08
Hills Herb-rich Woodland	71	15	150	7	641.42	0.872	6545.51
Creekline Herb-rich Woodland	164	15	150	7	281.36	0.383	2409.04
Grassy Woodland	175	5	45	7	141.44	0.192	1285.21
Valley Slopes Dry Forest	177	10	100	7	12.40	0.017	131.44
Sedgy Riparian Woodland	198	20	85	7	532.54	0.724	4946.06
Scoria Cone Woodland	894	4	15	7	20.74	0.028	219.84
Wet Forest	30	45	300	9	218.10	0.297	9396.53
Shrubby Wet Forest	201	25	150	9	825.47	1.123	34644.30
Riparian Forest	18	10	80	10	3.56	0.005	92.29
Swampy Riparian Woodland	83	15	125	10	1.89	0.003	43.65
Riparian Scrub or Swampy Riparian Woodland Complex	17	10	80	11	2561.76	3.484	30299.40
Wet Sands Thicket	233	15	90	11	27.27	0.037	370.87
Stream Bank Shrubland	851	15	90	11	38.32	0.052	521.15
Cool Temperate Rainforest	31	45	999	1	0.60	0.001	5.88
Wet Heathland	8	12	45	13	1416.63	1.926	18692.73
Damp Heath Scrub	165	10	90	13	1142.88	1.554	15908.60
Damp Heath Scrub/Heathy Woodland Complex	836	10	90	13	16.05	0.022	234.33
Sand Heathland	6	8	45	14	132.81	0.181	1684.73
Clay Heathland	7	10	45	14	30.58	0.042	405.60
Coastal Dune Scrub or Coastal Dune Grassland Mosaic	1	10	90	1	253.84	0.345	3016.53
Coastal Headland Scrub	161	8	90	1	1077.69	1.466	12587.77
Coastal Headland Scrub/Coastal Tussock Grassland Mosaic	162	8	90	1	98.98	0.135	1177.86
Coast Gully Thicket	181	10	90	1	1.67	0.002	15.52
Coastal Alkaline Scrub	858	10	70	1	11.82	0.016	140.65
Coastal Saltmarsh/Mangrove Shrubland Mosaic	302	8	90	2	4.52	0.006	14.46
Coastal Tussock Grassland	163	5	40	3	260.27	0.354	3773.91
Heathy Woodland	48	5	45	4	15985.16	21.738	313589.23
Shrubby Woodland	282	10	45	4	220.56	0.300	3465.91
Lowland Forest	16	8	80	5	21454.24	29.175	574823.49
Heathy Dry Forest	20	10	45	5	3958.52	5.383	95741.43
Shrubby Dry Forest	21	5	45	5	2299.87	3.128	64937.21
Grassy Dry Forest	22	5	45	6	2006.33	2.728	38475.14
Herb rich Foothill Forest	23	8	90	6	1670.13	2.271	34302.81
Shrubby Foothill Forest	45	8	90	6	12945.85	17.605	258807.84
Herb-rich Foothill Forest/Shrubby Foothill Forest Complex	178	8	90	6	2027.99	2.758	39253.237
Damp Sands Herb Rich Woodland	3	10	90	7	270.13	0.367	2776.23
Valley Grassy Forest	47	10	100	7	397.99	0.541	4054.89
Plains Grassy Woodland	55	4	15	7	482.38	0.656	4589.66
Alluvial Terraces Herb-Rich Woodland	67	4	15	7	56.07	0.076	594.34

Table 3.3: Total fuel load and solution time (seconds) or optimality gap (%) at 10800 seconds, for 40 treatment units with five, ten and 15-year planning horizon

	using the exact MIP approach		using the exact method for the single-period problem		using the approximate method for the single-period problem	
	treatment level		treatment level		treatment level	
	five percent	ten percent	five percent	ten percent	five percent	ten percent
WITH TFI						
5-year planning horizon						
solution time	0.28 sec	0.61 sec	6.36 sec	6.96 sec	0.01 sec	<0.01 sec
total fuel load (tonnes)	86485.18	68881.18	86760.76	69810.45	86719.71	73751.99
10-year planning horizon						
solution time	0.55 sec	3.18 sec	15.33 sec	11.59 sec	0.02 sec	0.01 sec
total fuel load (tonnes)	167073.99	126230.60	169480.60	130989.90	168897.70	134543.14
15-year planning horizon						
solution time	8.80 sec	7586.33	20.54 sec	16.06 sec	0.03 sec	0.01 sec
total fuel load (tonnes)	245671.97	185310.04	250616.10	193569.77	250348.40	195460.34
WITHOUT TFI						
5-year planning horizon						
solution time	0.84 sec	4.89 sec	6.94 sec	9.41 sec	0.01 sec	0.01 sec
total fuel load (tonnes)	85312.245	68445.858	85518.07	68752.81	85416.47	71016.17
10-year planning horizon						
solution time	307.87 sec	(0.58%)	13.46 sec	18.26 sec	0.02 sec	0.03 sec
total fuel load (tonnes)	163279.77	(123346.30)	165634.87	125142.26	165743.48	129400.07
15-year planning horizon						
solution time	(4.59 %)	(8.82 %)	20.58 sec	27.85 sec	0.03 sec	0.04 sec
total fuel load (tonnes)	(242727.28)	(180937.79)	245443.79	180974.32	246804.43	186476.47

Figure 3.3: Fuel load over time for 40 treatment units with a five-year planning horizon for the reduced study area

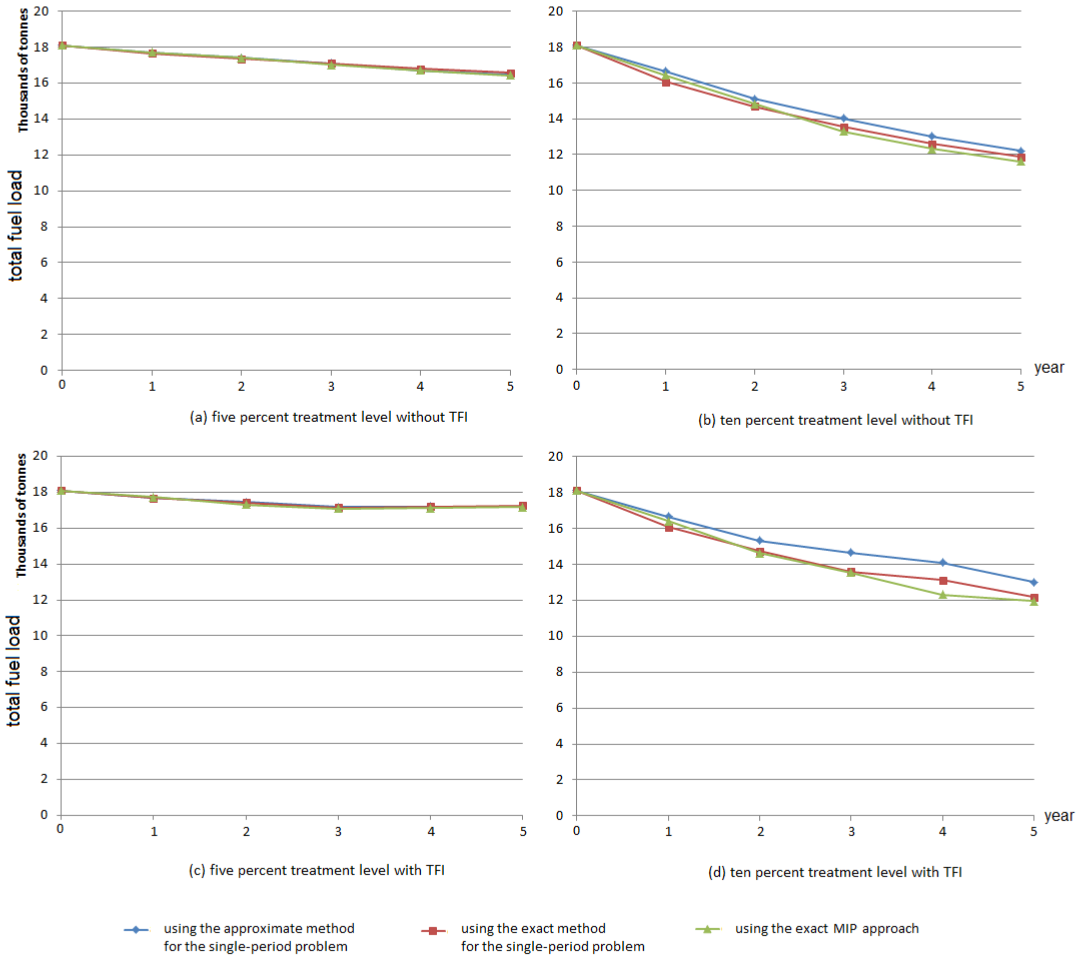


Figure 3.4: Fuel load over time for 40 treatment units with ten-year planning horizon, with TFI

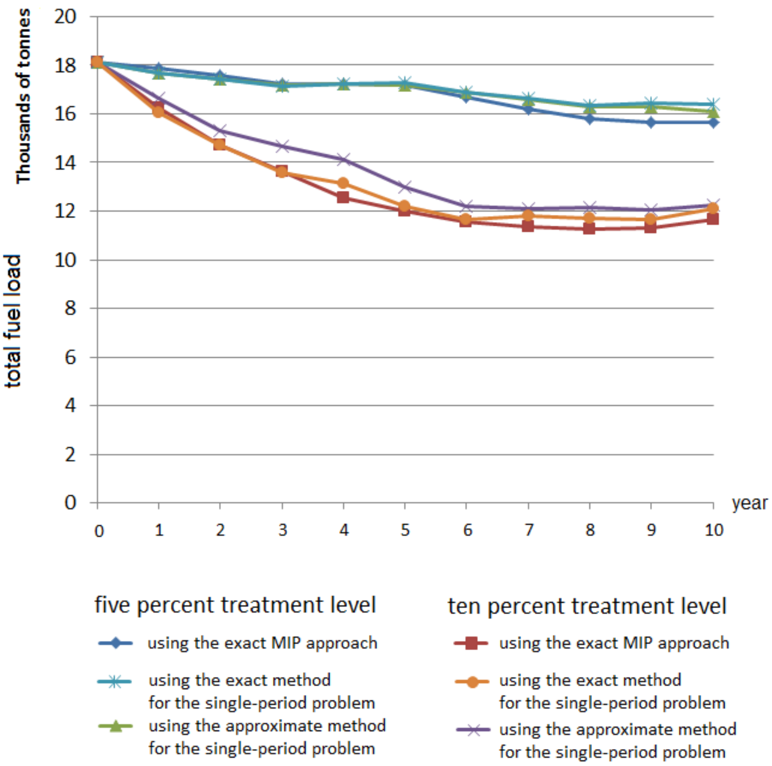
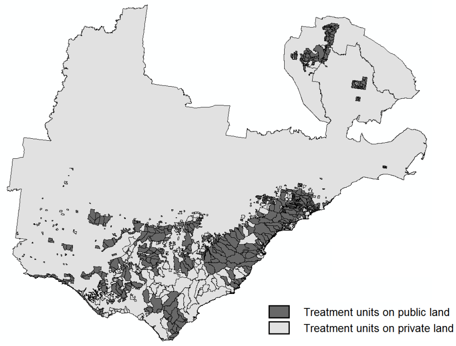


Figure 3.5: Location of the case study

(a) Location of the case study in the Barwon-Otway district of Victoria, Australia



(b) Map showing the distribution of treatment units within the Barwon-Otway case study area

Each vegetation type in this case study has its own fuel type and fuel accumulation loads over time as described in Figure 3.1. The curves show that each vegetation type has a different level of fuel load depending on age. In addition, there are some aquatic vegetation types or communities that have zero fuel loads and as such require no treatment. In this chapter those vegetation types are excluded. Table 3.2 lists the Ecological Vegetation Class (EVC) name and its fuel type used in this case study.

3.5.1 Using an exact Mixed Integer Programming (MIP) approach

There are two phases when using this approach for the case study. Phase 1 (an exact method for the single-period problem) is a preliminary stage before the Phase 2 approach (an exact MIP approach) is executed. In Phase 1, the 'old treatment units' in the landscape are identified. The purpose of Phase 1 is to handle any infeasibility that might arise based on the initial data, by burning the old treatment units. This phase is necessary for ensuring feasibility of the Phase 2 approach. Infeasibility may arise due to conflicting constraints, especially constraints (3.4), (3.7) and (3.8). Constraints (3.4) and (3.7) in the Phase 2 approach require that all 'old treatment units' must be treated. However, treating all of these old treatment units (in this case, 35 percent of the total treatable area in the landscape, as can be seen in Figure 3.6a) would violate Constraint (3.8) if the treatment level is set lower than 35 percent. In practice, it would also be costly and impractical to treat such a large amount of land in a single year. Moreover, The 2009 Victorian Bushfires Royal Commission nominates a target of five percent of the public land to be treated each year across the state in order to reduce the threat of fire for the coming fire season (Teague et al., 2010). Using a five percent treatment level across the case study area means that imposing the maximum TFI leads to infeasibility of the Phase 2 approach. Therefore, to reduce the number of 'old treatment units' and achieve feasibility first, in Phase 1 the treatment level must be increased. For Phase 1 of the case study, a treatment level of seven percent of the total area of the landscape each year is imposed. Interestingly, (Penman et al., 2011) note that when more than seven percent of the total area has been burnt by prescribed fire, the total area burnt by unplanned fire will be close to zero.

Phase 1 is solved for consecutive years using the solution of the previous year as an input until the problem is reconciled, containing less than five percent of 'old treatment units' in the landscape, as can be seen in Figure 3.6. Based on

Table 3.4: Computational comparison between the three model configurations using a 5% treatment level

Length of planning horizon	Solution time (seconds) or optimality gap (%) at 10800 seconds		
	'total'	'subset'	'random'
5 years	2.12	0.47	2.02
10 years	43.19	6.31	37.06
15 years	7819.6	46.23	3194.94
20 years	(0.27 %)	80.79	(0.04 %)
25 years	(5.45 %)	265.05	(1.02 %)

the initial data, it would take six years to achieve that for our case study. The model data, now feasible, enables us to move to Phase 2.

In Phase 2, the exact MIP approach is applied to ten-yearly planning horizons. The objective function is to minimise the total fuel load whilst meeting the constraints that have been described in subsection 3.3.1. Figure 3.7 represents the result of Phase 2 and identifies the location of treatments for each year to minimise the total fuel load while satisfying the minimum and maximum TFI constraints. The length of the planning horizon is ten years and the treatment level of each year is less than or equal to five percent.

The model is solved using ILOG CPLEX 12.6.2 with the Python 2.7 programming language. Computational experiments are performed on Trifid, a V3 Alliance high performance computer cluster. We tested the original problem

Figure 3.6: Solution of Phase 1

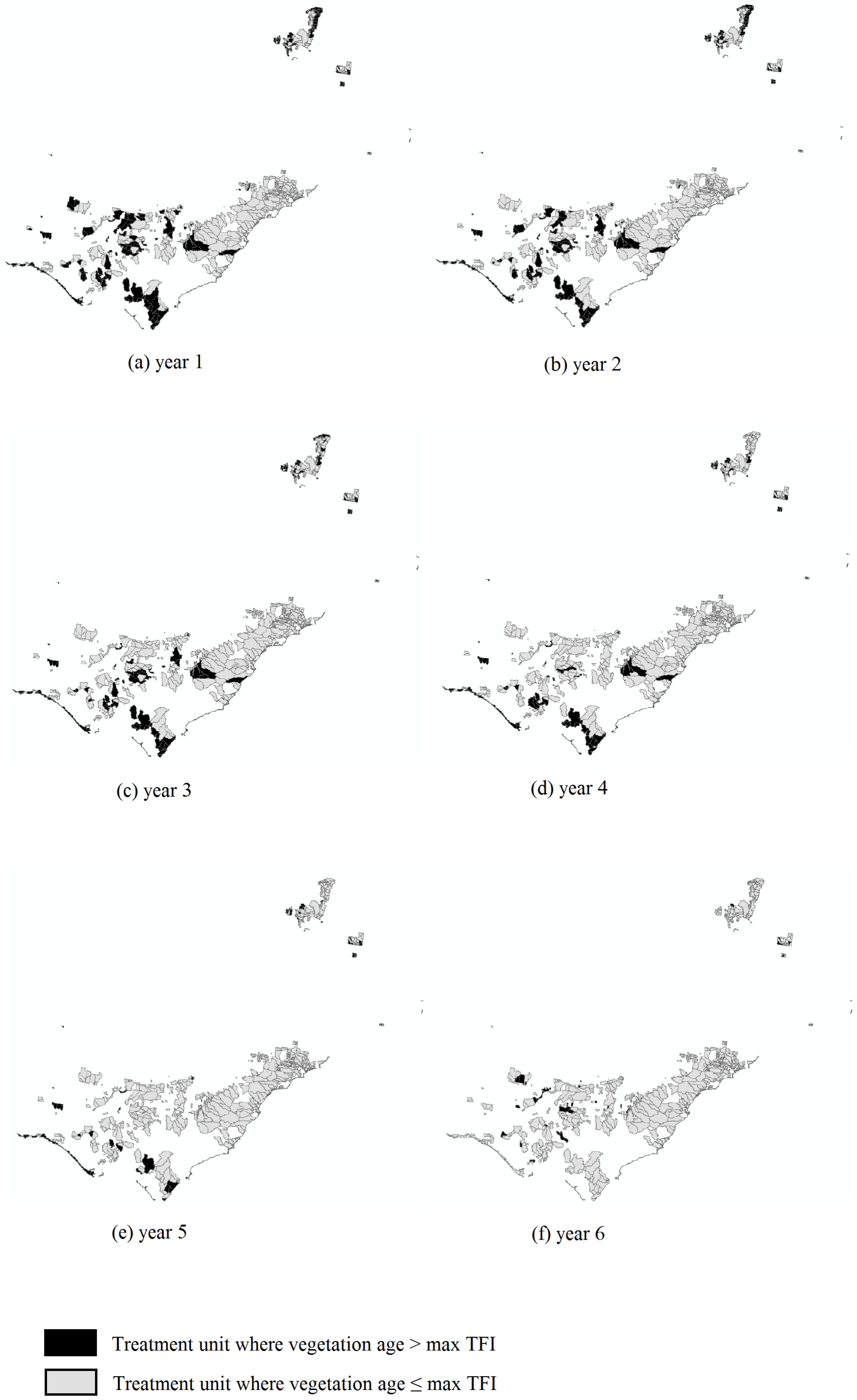


Figure 3.7: Solution of Phase 2

(a) year 1

(b) year 2

(c) year 4

(d) year 6

(e) year 8

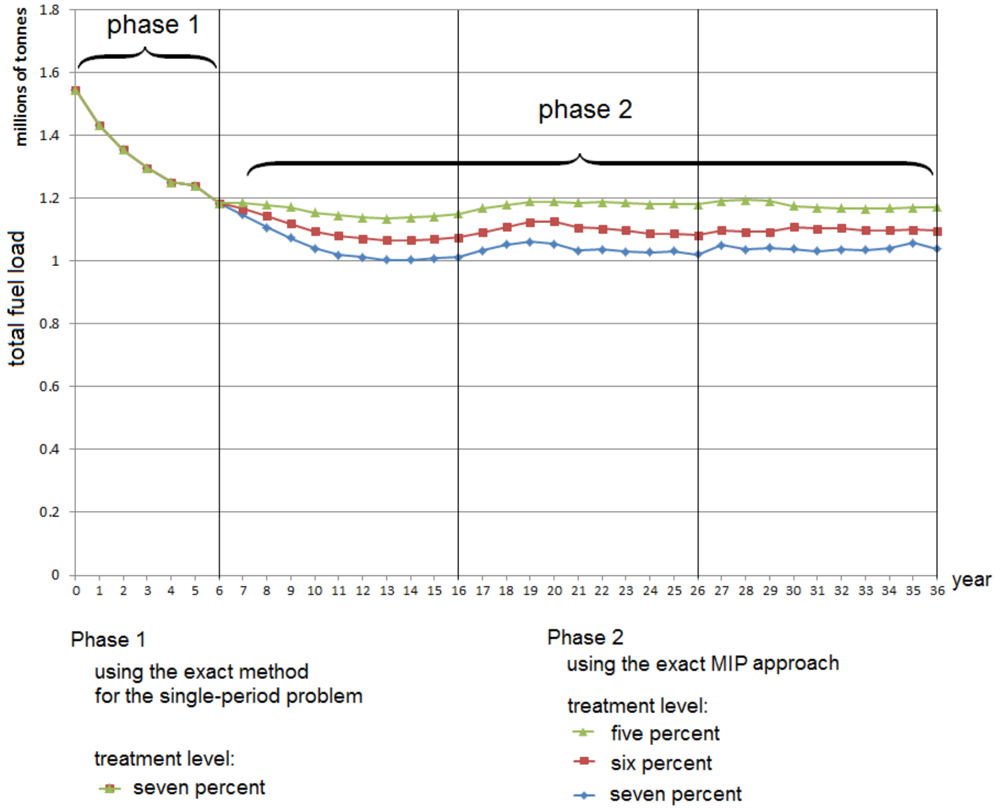
(f) year 10

and noticed the solution time of the relaxed problem [using constraint (3.14)] is no better than the original most likely due to valid inequalities introduced by CPLEX. Based on that, we decided to use the original problem, not its relaxed version.

Computation time against different model configurations is tested and the results are represented in Table 3.4. The CPU time or the gap between the best solution identified and the current linear programming relaxation is presented. The solution may actually be optimal but CPLEX may need a long time to prove it. The three model configurations are: 'total' (total fuel load where all treatment units are considered equal), 'subset' (total fuel load where a subset of treatment units are prioritised) and 'random' (total fuel load where random weights are assigned to treatment units). In 'total', all w_i's $= 1$. It means that the model minimises the total fuel load in all treatment units in the landscape, without prioritising certain regions. In 'subset', the value of $w_i = 1$ for some priority regions, and $w_i = 0$ for the other region. This priority may be due to proximity to towns. In 'random', $0 < w_i < 1$ assigned a relative importance weight to treatment unit which may be based on the population at risk or any other measure of defining relative importance.

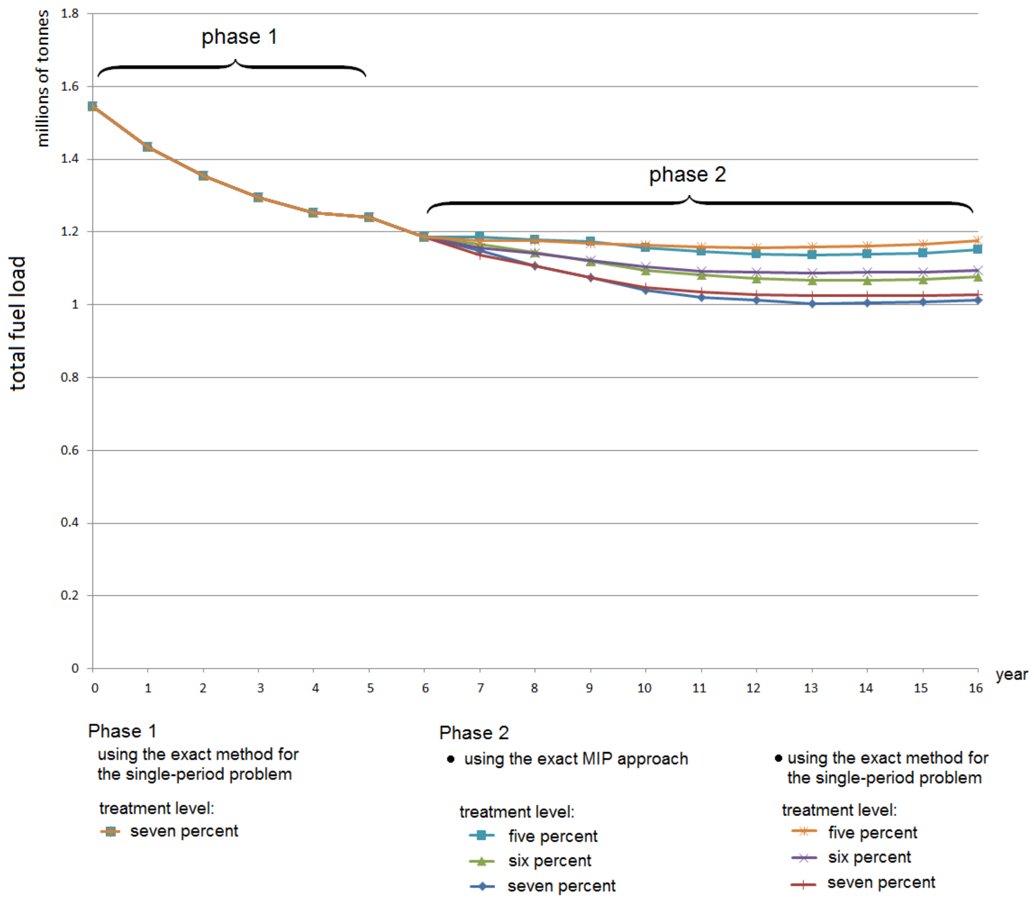
The optimal solutions of Phase 1 and Phase 2 are represented in Figure 3.6 and Figure 3.7, respectively. The solutions suggest where and when to conduct fuel treatments so as to minimise total fuel load accumulation. Figure 3.8 summarises the total fuel load over time for Phase 1 and Phase 2 for five, six and seven percent annual treatment levels. From the graph it is clear that the seven percent

Figure 3.8: Total fuel load over time



treatment level has the least total fuel load at every point in time, which is to be expected. However, a five percent treatment level has the most stable total fuel load in the long term. In other words, less variation is seen between years. For Phase 2 (i.e. from year 7 to 36), the approximate mean total fuel load and standard deviations in the landscape for five, six and seven percent treatment level are 1.171 million tonnes (standard deviation 17,000 tonnes), 1.099 million tonnes (standard deviation 21,000 tonnes) and 1.041 million tonnes (standard deviation 29,000), respectively.

Figure 3.9: Comparison of the total fuel load using the exact MIP approach and the exact method for the single-period approach



3.5.2 Using single-period heuristic approaches

Phase 2 can also be performed with the single-period heuristic approach for five, six and seven percent. In Phase 2, the exact MIP approach provides a slightly better optimal solution (less total fuel load) than that of the single-period heuristic approach, as can be seen in Figure 3.9. The differences between the exact MIP approach and the exact single-period heuristic approach for five, six and seven percent treatment levels are 0.93%, 0.94% and 1.02%, respectively. However, using the exact MIP approach for the longer planning horizon, e.g. 100 years, is very difficult, while using the exact single-period heuristic approach a relatively good solution can be achieved in a reasonable computational time ($<$ 3 minutes for 100-year planning). Because of their practicality, in the case study we then run the model with single-period heuristic approaches for 100 years.

Both approaches (an exact method for the single-period problem and an approximate method for the single-period problem) can directly be applied to the initial data. The result from both approaches is almost identical, because there are many small treatment units in the landscape so that the burn limit requirement can be met (almost) entirely.

In this case study, the computational experiments with or without incorporating TFI requirements are also conducted. The results for five and ten percent treatment level are represented in figures 3.10 and 3.11, respectively. By incorporating TFI requirements, when the treatment level is relatively high, e.g. ten percent annually, for some years the area burned may be less than ten percent

Figure 3.10: Total fuel load over time using the single-period heuristic approaches – five percent treatment level

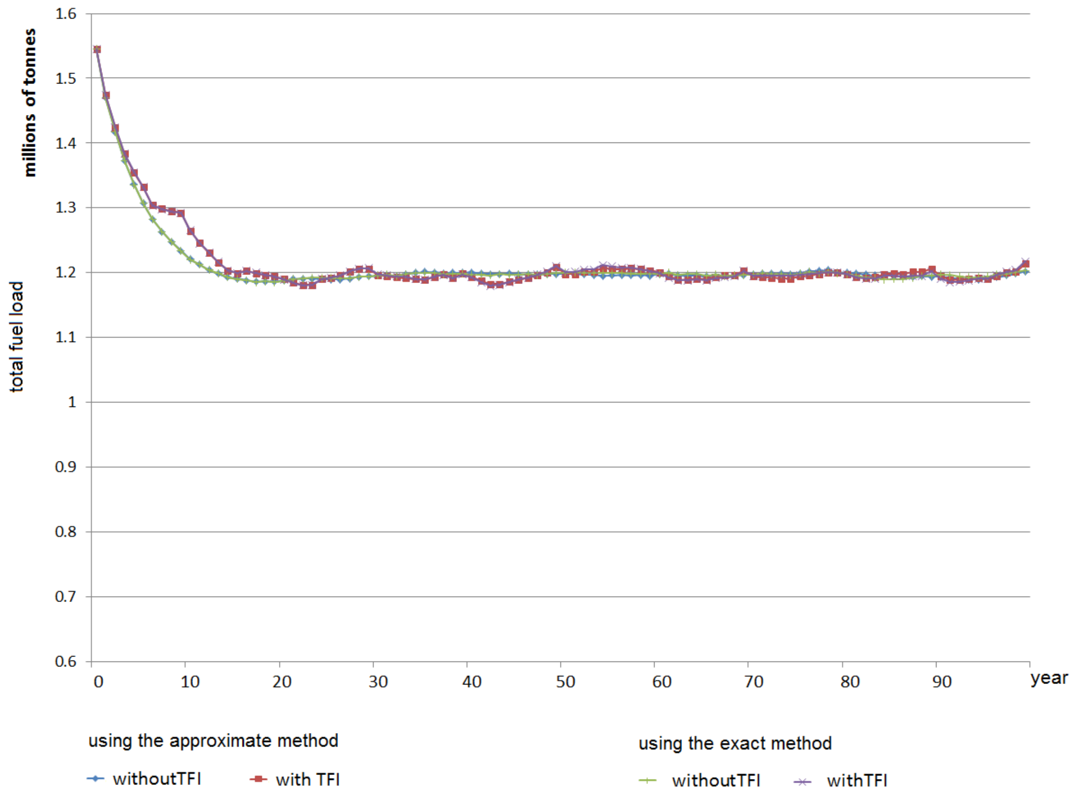
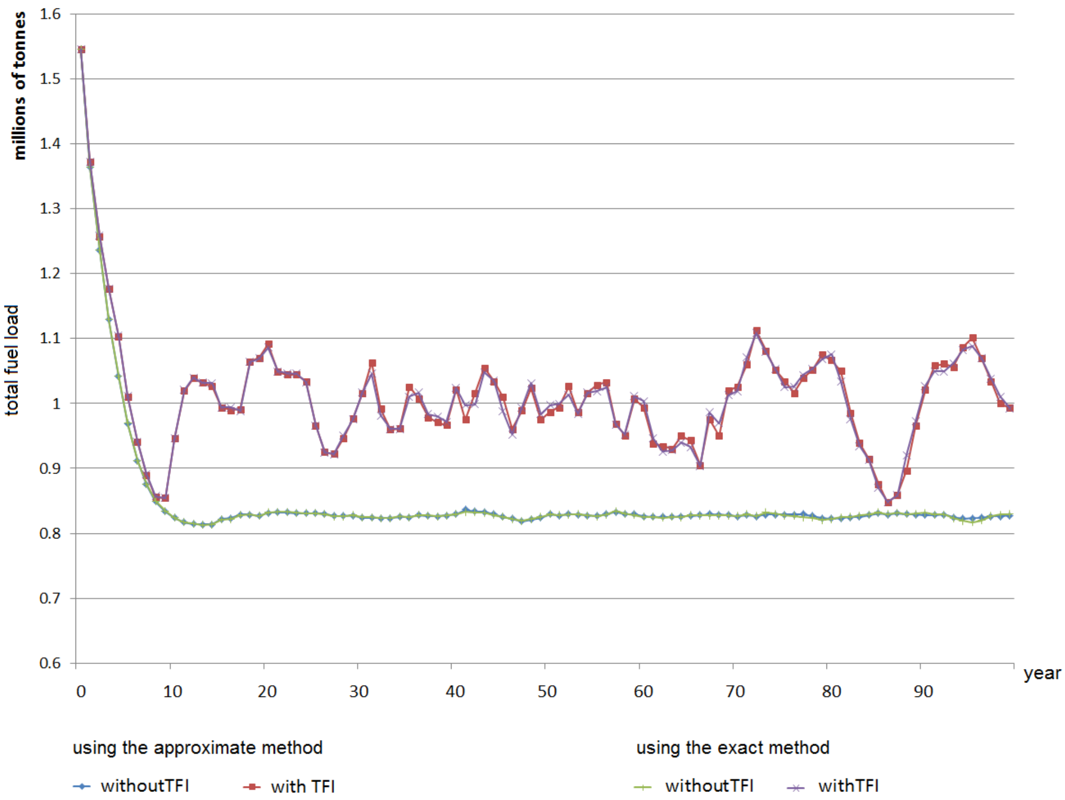


Figure 3.11: Total fuel load over time using the single-period heuristic approaches – ten percent treatment level



in subsequent years. This is because the vegetation needs some time to regrow until it is eligible to be treated. The treatment units can only be burned if all of the vegetation types in the treatment unit are above the minimum TFI. Figures 3.10 and 3.11 also represent the results of excluding the TFI requirement, which the total fuel load in the landscape is relatively very stable. However, due to the importance of TFI as discussed in Section 3.2, in practice excluding these requirements is not recommended.

3.6 Chapter summary

The purpose of this study was two-fold. Firstly, to develop an optimisation method for scheduling prescribed burns, and to embed this in a real-world case study that takes into consideration the spatial and temporal complexity of the problem. Secondly, to consider the fitness for purpose of our models by comparing the performance of simpler, heuristic-based solutions to a more complex, optimisation-based solution.

The complex multi-period model proposed takes into account multiple vegetation types of mixed ages in the landscape with differing nonlinear fuel accumulation functions. The model determines when and where to conduct fuel treatment to reduce the total fuel load in the landscape while still considering the ecological constraints relating to the Tolerable Fire Interval of each vegetation class. However, the spatial relationship between each treatment unit in the landscape is not considered in this chapter. In Chapter 4, we will particularly focus on breaking the connectivity of high fuel load areas to reduce fuel hazards in the landscape.

We compared the exact MIP and two heuristic approaches (Knapsack Problem and a greedy heuristic approach) in terms of the model tractability, computational time and the objective values. The solution for a ten-yearly planning horizon for the case study comprising 711 treatment units in the Barwon-Otway district of Victoria was obtained in 15 minutes by using the exact MIP approach. With longer time periods it was not possible to achieve solutions of sufficient accuracy within a few days. While this approach can provide optimal solutions, it is computationally costly, especially for fuel management and ecological planning which may require longer planning horizons, and cover much larger geographic areas. Meanwhile, the heuristic methods can solve the problem for a longer times (e.g., 100 years), and the solution can be obtained in less than three minutes.

Based on our experiments, the single-period decomposition works well, and Knapsack MIP performs almost as well as the multi-period MIP. For a ten-year planning horizon with five, six and seven percent treatment levels, the objective values resulting from these two approaches differs approximately by only one percent. It is clear from the series of computational experiments that the solutions resulting from the heuristic approaches mimic that of the exact MIP to solve the prescribed burn planning model. The case study shows that the heuristic approaches provide a near-optimal solution and the computational time is significantly faster than that of the exact MIP approach. We conclude that for practical purposes a heuristic method is more than adequate.

4 An optimisation approach for fuel treatment planning to break the connectivity of high-risk regions

In the previous chapter we have shown a model to lessen the risk of fire by reducing the total fuel load but do not consider the spatial relationship between the treatment units. In Section 1.3, we have discussed the importance of fragmenting high-risk areas. Therefore, in this chapter we formulate an approach to reduce the spatial connectivity of fuel hazards.

4.1 Introduction

The choice of fuel treatment location plays a substantial role in conducting efficient fuel treatment scheduling (Collins et al., 2010). Instead of randomly selecting the locations, significantly better protection in a landscape could be provided by a fuel treatment schedule that takes into account the relationships between treatment units (Schmidt et al., 2008). Research indicates that it is important to choose where to conduct the fuel treatment by considering spatial arrangement (Rytwinski and Crowe, 2010; Kim et al., 2009; Chung, 2015). The importance of landscape-level fuel treatment has been observed in a number of studies. In wilderness regions in the United States, a mosaic of varying fuel ages is formed as a result of free burning fires. A particular arrangement of old and new treatment units has been recognised to delay large wildfires in the following year (Finney, 2007). Research conducted in the Sierra Nevada forests of the United States has shown that wildfire size can be modified by spatial fragmentation of fuel (Van Wagtendonk, 1995). Prescribed burning has been implemented in the eucalypt forests in south-western Australia over the past 50 years. The connectivity of 'old' untreated patches has been revealed to be the main aspect that contributes to wildfire extent (Boer et al., 2009).

Previous studies have proposed mathematically modelling fuel treatment schedules methods for reducing fuel hazards. The studies had different objective functions and took into account various considerations in building up the models. Ferreira et al. (2014) proposed a stochastic dynamic programming (SDP) approach to determine the fuel treatment scheduling that produces the maximum expected discounted net revenue while mitigating the risk of fire. The method was then applied to a maritime pine forest in Leiria National Forest, Portugal. They found that the approach was efficient and can efficiently help integrating wildfire risk in stand management planning. Garcia-Gonzalo et al. (2014) used the Hooke-Jeeves direct search method to determine the optimal fuel treatment scheduling for reducing expected damage and increasing the revenue to the same landscape, as that of Ferreira et al. (2014). Their research shows that the fuel treatments improve productivity as well as reduce the potential damage. Rachmawati et al. (2015) proposed a model to reduce the risk of fire by minimising the total fuel load in a landscape but do not consider spatial properties or the spatial relationship between the treatment units. Wei and Long (2014) proposed a single-period model to fragment high-risk patches by considering future fire spread speeds and durations. Minas et al. (2014) proposed a model that breaks the connectivity of high fuel units in the landscape to prevent the fires spreading. The model proposed by Minas et al. (2014) takes into account vegetation dynamics in the landscape, but this is limited to a simplistic grid representation of a single vegetation type per treatment unit. In reality, a treatment unit may comprise a number of patches with different vegetation type and age. In summary, most of the models reviewed can be improved by taking into account multi-vegetation types and ages within a treatment unit and using a polygon-based network representation.

In this chapter, we build upon previous work by incorporating multiple vegetation types found in the landscape and within single treatment units, and take into account the spatial connectivity or fragmentation of 'high-risk' treatment units. We also use a more realistic polygon-based network representation of the landscape to better capture the spatial complexity of this problem rather than a rectangular grid. Besides the negative impacts of wildfires, the role of fire in ecology has been widely acknowledged. Fire is required to maintain a healthy ecosystem and it also has a significant role in habitat regeneration. Many vegetation species in fire-adapted ecosystems need fire to reproduce. For instance, germination of seeds and successful establishment of plants in the jarrah forests of Western Australia is very rarely found without fire intervention (Burrows and Wardell-Johnson, 2003). More recently, Burrows (2008) argued that fuel management is important to support biodiversity conservation as well as to reduce the negative impact of wildfires. A recognition of vegetation dynamics over time is crucial in the planning of fuel treatment (Krivtsov et al., 2009). In this proposed model, similar to the previous chapter, the ecological fire requirements of each vegetation type can be described using the minimum and maximum Tolerable Fire Intervals (TFI). We assume that treatment of vegetation whose age is between these two intervals will maintain species diversity and hence support the ecosystem's health. Therefore, we select not to treat a treatment unit if the age of vegetation growing in that location is under the minimum TFI. In contrast, treatment units with vegetation over the maximum TFI must be treated. In this chapter, we assume that the high-risk threshold age is between these two intervals. The objective of the model proposed in this chapter is to reduce the spatial connectivity of fuel hazards while still considering the fire requirements of the ecosystem. The question that then arises is when and where to conduct fuel treatment to meet this objective, that can be solved for spatially complex landscapes with long planning horizons?

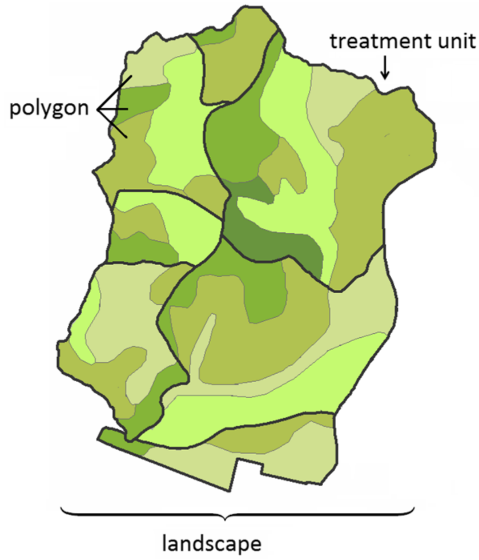
A Mixed Integer Programming (MIP) model is proposed for multi-period fuel treatment scheduling. The model tracks the vegetation age in each treatment unit yearly for both treated and untreated areas. The model is then applied to a real landscape in southern Australia that comprises different shapes and sizes of treatment units.

4.2 Problem formulation

In this section, we explain the terms ‘treatment unit’ and ‘patch’ that we use to formulate the problem. The candidate locations for fuel treatment are represented by treatment units. A treatment unit comprises multiple patches (Figure 4.1). Each vegetation type growing in a treatment unit is represented by a patch and within each patch all the vegetation is of the same age. The data in each patch includes area, vegetation type and age. Patches within a single treatment unit may have different vegetation type and age, defining a ‘multi-vegetation treatment unit’.

Each vegetation type has a ‘high-risk’ age threshold. For example, grass and bush are considered to be high-risk when they reach four and seven years old, respectively. Since we know the vegetation type and age in each patch, we then know whether a patch is a high-risk patch or not at any given time. In order to disconnect the high-risk treatment units in a landscape, we need a method to determine whether a treatment unit is a high-risk treatment unit or not. In this chapter, we assume that if ignitions occur, the fires will likely spread through connected high-risk treatment units. From this, we believe that if we

Figure 4.1: A landscape, divided into treatment units, sub-divided into patches



can disconnect high-risk treatment units as much as possible, the possibility of catastrophic fires can be reduced.

Each treatment unit selected for fuel treatment should not violate the ecological requirements. Each vegetation type has its specific minimum and maximum TFI. We assume that a healthy ecosystem can be maintained when the fuel treatment is conducted when the vegetation age is between the minimum and the maximum TFI.

4.3 Model formulation

The model is formulated to determine when and where to conduct the fuel treatment each year to break the connectivity of high-risk treatment units and to meet the ecological requirements. We consider a landscape divided into treatment units where each treatment unit might consist of multiple patches. The following mixed integer programming model is formulated.

Sets:

C is the set of all treatment units in the landscape

$\Psi \subset C$ is the set of treatment units where fuel treatment is not permitted

$\Lambda \subset C$ is the set of treatment units where fuel treatment is permitted (where $\Lambda = C - \Psi$)

P_i is the set of patches in treatment unit i

Φ_i is the set of treatment units connected to treatment unit i

T is the planning horizon

Indices:

p = patch

i = treatment unit

t = period, $t = 0, 1, 2, \ldots T$

Parameters:

$w_{i,j}$ = relative importance (weight) of connectivity of treatment units i and j

a_p = initial vegetation age in patch p

$Area_p$= area of patch p

ρ = treatment level (in percentage), i.e. the maximum proportion of the total area that fuel treatment is permitted in a landscape selected for treatment

R = the total area of treatment units in the landscape where fuel treatment is permitted

c_i= area of treatment unit i

d_p = high-risk age threshold for patch p, based upon the vegetation type growing in that patch

$maxTFI_p$= maximum tolerable fire interval (TFI) of vegetation type growing in patch p

$minTFI_p$ = minimum TFI of vegetation type growing in patch p

H = the threshold for the area proportion of the high-risk patches in a treatment unit to be a high-risk treatment units

Decision variables:

$A_{p,t}$ = vegetation age in patch p at time t

$$x_{i,t} = \begin{cases} 1 & \text{if treatment unit } i \text{ is treated in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$Highpatch_{p,t} = \begin{cases} 1 & \text{if patch } p \text{ is classified as high-risk patch in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$High_{i,t} = \begin{cases} 1 & \text{if treatment unit } i \text{ is classified as high-risk} \\ & \text{treatment unit in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$HighConn_{i,j,t} = \begin{cases} 1 & \text{if connected treatment units } i \text{ and } j \text{ are both} \\ & \text{high-risk treatment units in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$Old_{p,t} = \begin{cases} 1 & \text{if patch } p \text{ is classified as `old' (over-the-maximum-TFI)} \\ & \text{patch in time period } t \\ 0 & \text{otherwise} \end{cases}$$

$$Young_{p,t} = \begin{cases} 1 & \text{if patch } p \text{ is classified as `young' (under-the-minimum-TFI)} \\ & \text{patch in time period } t \\ 0 & \text{otherwise} \end{cases}$$

Minimise the weighted connectivity of high-risk treatment units

$$z = \sum_{t=1}^{T} \sum_{i \in C} \sum_{j \in \Phi_i, i<j} w_{i,j} HighConn_{i,j,t} \tag{4.1}$$

subject to

$$\sum_i c_i x_{i,t} \leq \rho R,\ t = 1 \dots T, \forall i \in \Lambda \tag{4.2}$$

$$A_{p,0} = a_p,\ \forall p \tag{4.3}$$

$$A_{p,t} = A_{p,t-1} + 1,\ \forall p \in P_i, t = 1 \dots T, \forall i \in \Psi \tag{4.4}$$

$$A_{p,t} \geq A_{p,t-1} + 1 - M_1 x_{i,t},\ \forall p \in P_i, t = 1 \dots T, \forall i \in \Lambda \tag{4.5}$$

$$A_{p,t} \leq M_2(1 - x_{i,t}),\ \forall p \in P_i, t = 1 \dots T, \forall i \in \Lambda \tag{4.6}$$

$$A_{p,t} \leq A_{p,t-1} + 1,\ \forall p \in P_i, t = 1 \dots T, \forall i \in \Lambda \tag{4.7}$$

$$A_{p,t} - d_p \leq M_3 Highpatch_{p,t} - 1,\ \forall p \in P_i, t = 1 \dots T, \forall i \in C \tag{4.8}$$

$$\sum_{p \in P_i} Area_p Highpatch_{p,t} - H \sum_{p \in P_i} Area_p \leq M_4 High_{i,t},\ t = 1 \dots T, \forall p \in P_i, \forall i \in C \tag{4.9}$$

$$High_{i,t} + High_{j,t} - HighConn_{i,j,t} \leq 1,\ t = 1 \dots T, \forall j \in \Phi_i, i < j, \forall i \in C \tag{4.10}$$

$$A_{p,t} - maxTFI_p \leq M_5 Old_{p,t} - 1,\ \forall p \in P_i, t = 0 \ldots T-1, \forall i \in \Lambda \tag{4.11}$$

$$A_{p,t} \geq maxTFI_p Old_{p,t},\ \forall p \in P_i, t = 0 \ldots T-1, \forall i \in \Lambda \tag{4.12}$$

$$A_{p,t} + M_6 Young_{p,t} \geq minTFI_p,\ \forall p \in P_i, t = 0 \ldots T-1, \forall i \in \Lambda \tag{4.13}$$

$$A_{p,t} - M_7(1 - Young_{p,t}) \leq minTFI_p - 1,\ \forall p \in P_i, t = 0 \ldots T-1, \forall i \in \Lambda \tag{4.14}$$

$$Young_{p,t-1} \leq 1 - x_{i,t},\ t = 1 \ldots T, \forall i \in \Lambda \tag{4.15}$$

$$\sum_{p \in P_i} Old_{p,t-1} - \mid V_i \mid \sum_{p \in P_i} Young_{p,t-1} \leq \mid V_i \mid x_{i,t},\ t = 1 \ldots T, \forall i \in \Lambda \tag{4.16}$$

$$x_{i,t},\ Highpatch_{p,t},\ High_{i,t},\ HighConn_{i,j,t},\ Young_{p,t},\ Old_{p,t} \in \{0,1\} \tag{4.17}$$

The objective function (4.1) minimises the weighted connectivity of high-risk treatment units in a landscape throughout a planning horizon.

Constraint (4.2) specifies that the total area selected for fuel treatment annually is not more than the area allotted (target) each year for fuel treatment (in hectares).

Constraint (4.3) sets the initial vegetation age in a patch. Constraint (4.4) to (4.6) track the vegetation age of each patch. Constraint (4.4) relates to the set of treatment units where fuel treatment is not permitted. Constraint (4.5) and (4.6) indicate that when $x_{i,t} = 0$, the vegetation in that area will continue growing until the following period, and the age will be incremented by one. Whereas if $x_{i,t} = 1$, the vegetation age will reset to zero. Constraint (4.7) increments vegetation age by exactly one year if the treatment unit is not treated.

Constraint (4.8) uses binary variable $Highpatch_{p,t}$ to classify a patch to be a high-risk patch if the vegetation age in that patch reaches or exceeds a threshold value, thus each patch has its own age threshold. Then, within a single treatment unit, we can compare the area of over-the-threshold patch. Here, we define a treatment unit as a high-risk treatment unit if the proportion of the over the threshold area is greater than a certain proportion of the total treatable area of the treatment unit. Constraint (4.9) represents this requirement. In constraint (4.10), $HighConn_{i,j,t}$ takes the value one if connected treatment units i and j are both classified as high-risk treatment units in time period t .

Constraints (4.11) to (4.14) classify a patch to be an 'old' or a 'young' patch based on TFI values. Constraint (4.15) ensures that the treatment units containing young patches cannot be treated. Constraint (4.16) states that if there is at least one patch within a treatment unit that is 'old' and no young patch, then the treatment unit must be treated. Here, $|V_i|$ represents the number of patches in treatment unit i. This constraint avoids a deadlock that may occur when a treatment unit consists of a young and an old patch at the same time. In this study, we break the deadlock in favour of young patch.

The M's coefficient in equations (4.5), (4.6), (4.8), (4.9), (4.11), (4.13) and (4.14) represent arbitrarily large Big-M.

Constraints (4.17) ensures that the decision variables take binary values.

4.3.1 Model improvements

The solution time can be improved by reducing the number of variables. As discussed earlier, the initial age of each vegetation type in each treatment unit is given. We also assume that the age of vegetation type growing in the treatment units where fuel treatment is not permitted should always be incremented by one. For this reason, we no longer need constraint (4.4) to track the vegetation in the area. The time for the vegetation type to reach the high-risk age threshold can be determined. And because we assume that we cannot treat the treatment units, once the vegetation type hits the threshold it will remain high risk. Therefore, within a planning horizon we can determine whether a treatment unit is high risk or not.

Decision variables $A_{p,t}$ and $Highpatch_{p,t}$ for the treatment units where fuel treatment is not permitted can be omitted, and regarded as parameters instead. This results in a faster solution time.

We can rewrite our model as follows. Constraint (4.4) is excluded, because at any given time the age of vegetation growing in the treatment units where fuel treatment is not permitted is known. Constraints (4.8) and (4.9) are only defined for treatable treatment units. All other constraints remain the same. However, we introduce these two constraints to the model for the treatment units where fuel treatments are not permitted:

$$High_{i,t} = 0, \ \forall t \text{ when } \theta \leq 0, \forall i \in \Psi \tag{4.18}$$

$$High_{i,t} = 1, \ \forall t \text{ when } \theta > 0, \forall i \in \Psi \tag{4.19}$$

where $\theta = \sum_{p \in P_i} Area_p Highpatch_{p,t} - H \sum_{p \in P_i} Area_p$

In constraint (4.18), value 0 is assigned to $High_{i,t}$ if less than a certain proportion of the total treatable area of the treatment unit is high risk at time t. And in constraint (4.19) value 1 is assigned to $High_{i,t}$ if more than a certain proportion of the total treatable area of the treatment unit is high risk at time t.

4.4 Implementation of the new approach

Initially, it may not be possible to treat all treatment units according to the maximum TFI value because of the annual limit, ρ. This maximum TFI requirement may lead to the infeasibility of the initial problem. In order to bring the system under control and to avoid the initial infeasibility, we propose a preliminary stage, namely Phase 1. From the initial data, we can identify treatment units containing an old patch or would potentially be containing an old patch in the following year and have no young patches. We are trying to eliminate the treatment units containing old patches to ensure feasibility. In this phase, we exclude the TFI constraints, which are constraints (4.11) to (4.16). We run the model without enforcing the constraint ensuring treatment of old patches for some years, and modify the objective function as follows:

maximise

$$z = \sum_{t=1}^{N}\sum_{i\in\Theta} c_i x_{i,t} - \sum_{t=1}^{N}\sum_{i\in\Theta}\sum_{j\in\Phi_i, i<j} \varepsilon_i HighConn_{i,j,t} \tag{4.20}$$

where Θ is the set of treatment units that contains an old patch or potentially contains an old patch in the following year and no young patch. ε_i is a relatively small number ($\varepsilon_i \ll c_i$) representing the weight of connectivity of treatment unit i. N is the planning horizon.

The objective is to maximise the area treated and to minimise the weighted connectivity of the treatment units in a landscape for a number of years ahead. The planning horizon (N) increased incrementally until the initial problem is feasible.

For the landscape that comprises mostly old treatment units, the solution

Table 4.1: Vegetation type and the associated threshold age, the minimum and the maximum TFI for the test landscape

vegetation type	min TFI (year)	max TFI (year)	threshold (year)
1	3	10	5
3	4	15	7
6	7	20	10

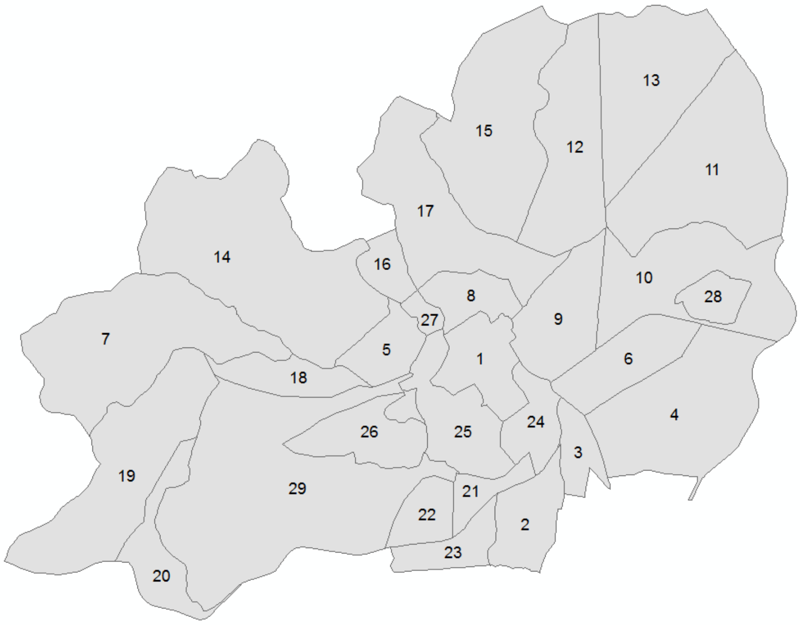
from this phase becomes the input for Phase 2. In Phase 2, the model presented in Section 4.3 is run.

4.5 Model demonstration

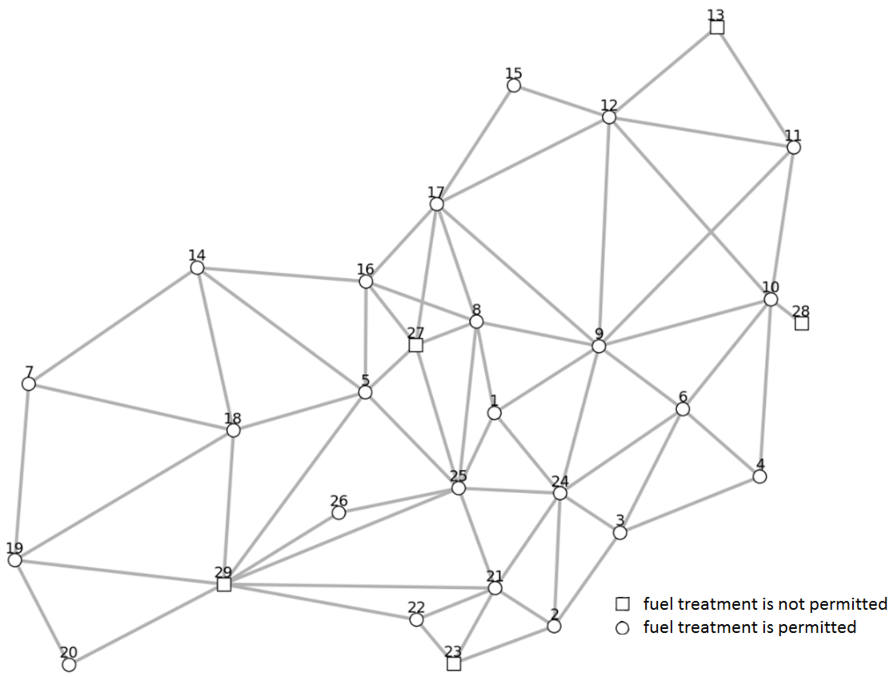
For the model demonstration, consider a test landscape comprising 29 treatment units that are a subset of the case study in the Barwon-Otway district of Victoria, Australia. Figure 4.3a represents the map of the landscape and Figure 4.3b illustrates the graph representing the neighbourhood of each treatment unit. We assume that two treatment units are neighbouring if they have common boundaries. Table 4.1 represents data for each vegetation type and the associated threshold age, the minimum and the maximum TFI for this test landscape. The data regarding the area of the treatment units, vegetation type and age can be seen in Table 4.2.

We evaluate the test landscape based on the data from Table 4.2. The rule is that if more than 50 percent of the treatment unit are high-risk patches, then we consider it as a high-risk treatment unit. Figure (4.3) and (4.4) show the network and the related map representing the fuel treatment schedule with 15

Figure 4.2: A landscape for the model demonstration (29 treatment units)

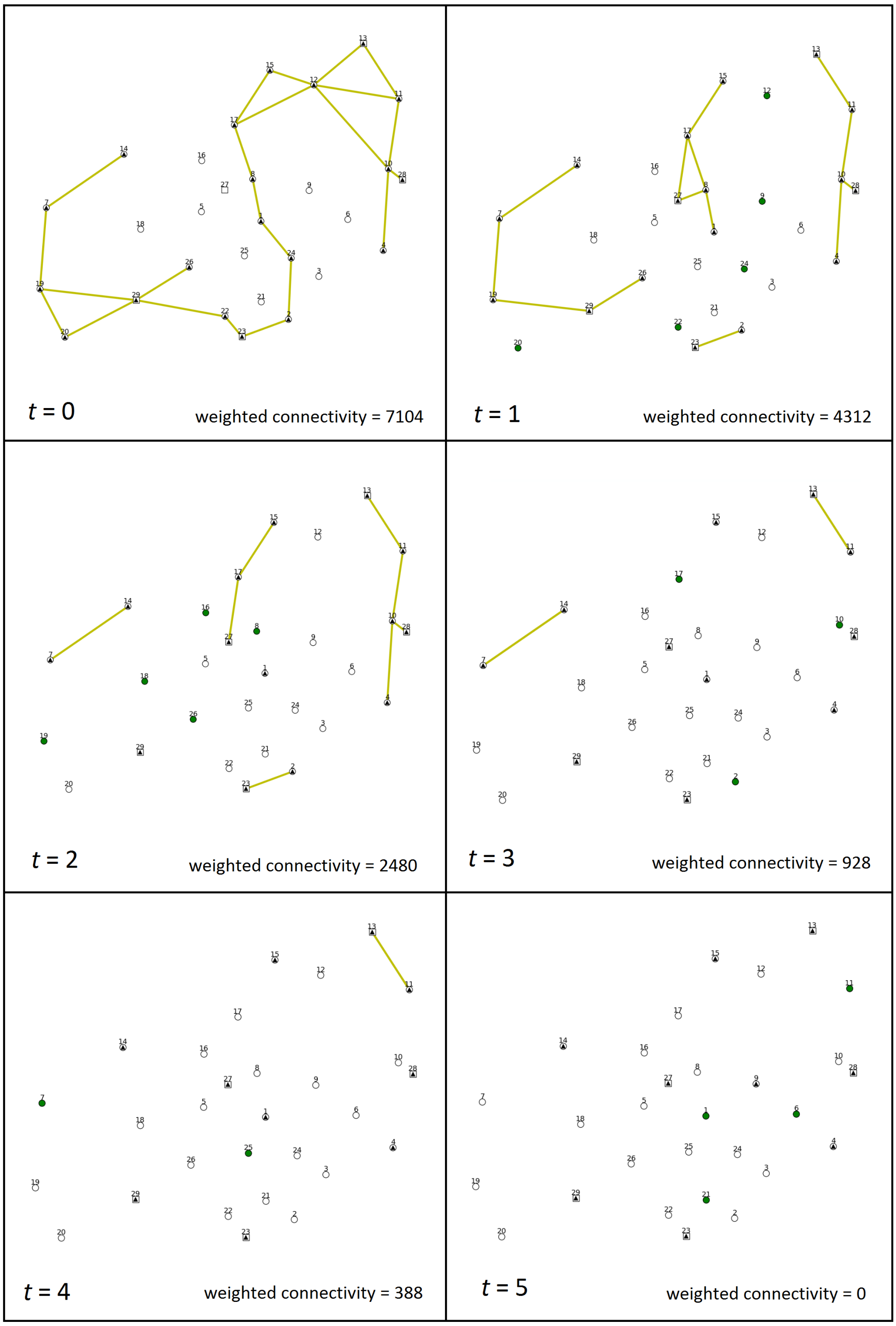


(a) Map of the landscape



(b) The neighbourhood graph of the landscape

Figure 4.3: A network represents the fuel treatment schedule for the test landscape



- Treatment unit selected for fuel treatment
- Low-risk treatment unit where fuel treatment is permitted
- Low-risk treatment unit where fuel treatment is not permitted
- High-risk treatment unit where fuel treatment is permitted
- High-risk treatment unit where fuel treatment is not permitted

Figure 4.4: The sequence of maps representing the fuel treatment schedule (in years) for the test landscape

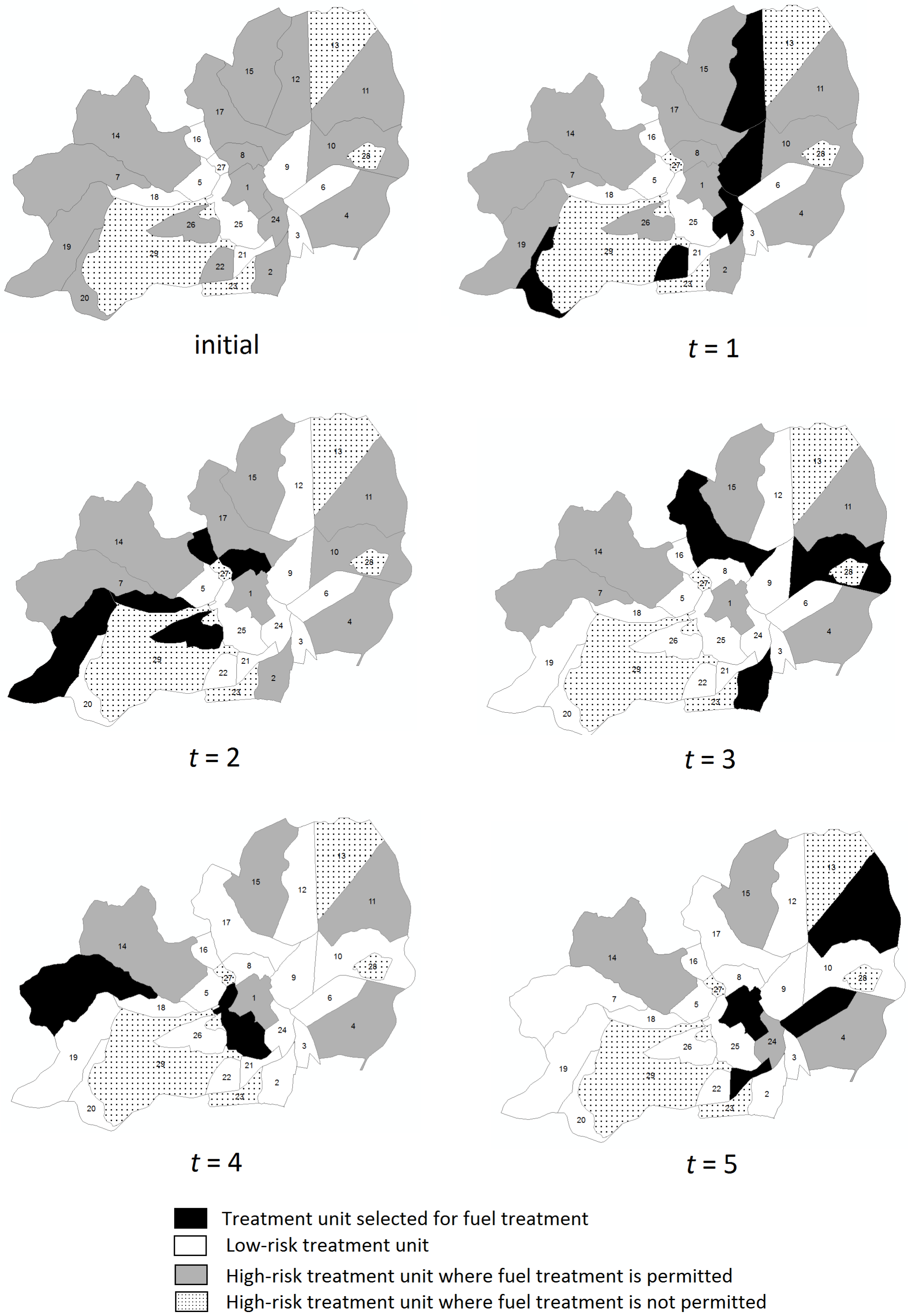


Table 4.2: 29 treatment units data containing vegetation type, extent and age

Treatment unit ID	veg type	area (ha)	age (years)	Treatment unit ID	veg type	area (ha)	age (years)	Treatment unit ID	veg type	area (ha)	age (years)
1	1	10	6	8	1	19	5	19	1	50	5
1	3	8	7	8	3	12	7	19	3	37	5
1	6	14	11	9	1	46	4	20	1	10	1
2	1	10	5	10	1	78	6	20	3	6	2
2	3	21	8	11	1	30	4	20	6	14	10
3	1	4	1	11	3	50	8	21	1	5	1
3	3	5	1	11	6	30	12	21	3	8	1
3	6	7	1	12	1	40	5	22	3	19	7
4	1	40	5	12	3	34	7	23	6	20	11
4	3	30	6	13	6	84	11	24	6	22	10
4	6	24	10	14	3	80	7	25	1	42	1
5	1	8	1	14	6	76	11	26	3	33	7
5	3	10	1	15	6	103	12	27	3	6	6
5	6	4	1	16	3	14	5	28	1	14	5
6	1	18	1	17	1	50	5	29	1	100	5
6	3	20	1	17	3	32	6	29	3	50	6
7	3	80	8	18	3	14	5	29	6	41	9
7	6	34	11	18	6	10	9				

percent treatment level, starting from the $t = 0$ which represents the initial condition of the landscape. We can treat the surrounding treatment units to break the connectivity of high-risk units. When the patch within a treatment unit has reached the maximum TFI, and no patch is below the minimum TFI, the treatment units should be treated. This ecological requirement applies even for the treatment units that do not contribute to the connectivity of high-risk areas.

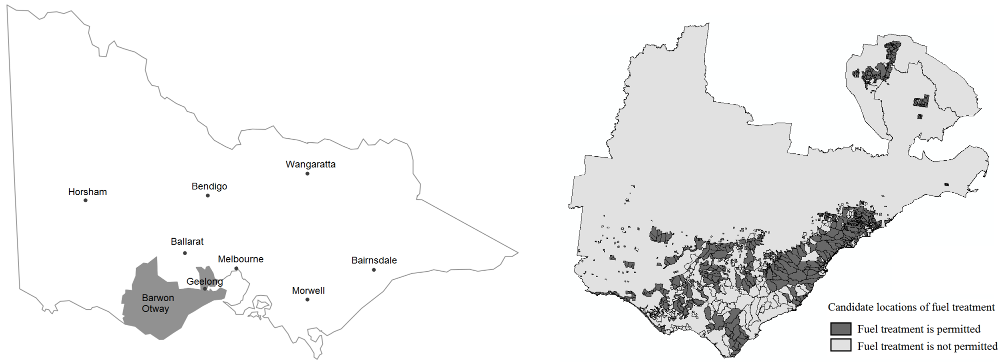
4.6 An Australian case study

In this section, we apply the model discussed in Section 4.3 to an Australian

Table 4.3: Ecological Vegetation Class (EVC) and the associated threshold age, the minimum and the maximum TFI

EVC name	min TFI (year)	max TFI (year)	threshold (year)
Creekline Grassy Woodland	20	150	20
Hills Herb-rich Woodland	15	150	17
Creekline Herb-rich Woodland	15	150	17
Grassy Woodland	5	45	17
Valley Slopes Dry Forest	10	100	17
Sedgy Riparian Woodland	20	85	20
Scoria Cone Woodland	4	15	15
Wet Forest	45	300	45
Shrubby Wet Forest	25	150	25
Riparian Forest	10	80	22
Swampy Riparian Woodland	15	125	22
Riparian Scrub or Swampy Riparian Woodland Complex	10	80	16
Wet Sands Thicket	15	90	16
Stream Bank Shrubland	15	90	16
Cool Temperate Rainforest	45	999	45
Wet Heathland	12	45	12
Damp Heath Scrub	10	90	10
Damp Heath Scrub/Heathy Woodland Complex	10	90	10
Sand Heathland	8	45	8
Clay Heathland	10	45	10
Coastal Dune Scrub or Coastal Dune Grassland Mosaic	10	90	17
Coastal Headland Scrub	8	90	17
Coastal Headland Scrub/Coastal Tussock Grassland Mosaic	8	90	17
Coast Gully Thicket	10	90	17
Coastal Alkaline Scrub	10	70	17
Coastal Saltmarsh/Mangrove Shrubland Mosaic	8	90	14
Coastal Tussock Grassland	5	40	6
Heathy Woodland	5	45	35
Shrubby Woodland	10	45	35
Lowland Forest	8	80	20
Heathy Dry Forest	10	45	20
Shrubby Dry Forest	5	45	20
Grassy Dry Forest	5	45	15
Herb rich Foothill Forest	8	90	15
Shrubby Foothill Forest	8	90	15
Herb-rich Foothill Forest/Shrubby Foothill Forest Complex	8	90	15
Damp Sands Herb Rich Woodland	10	90	17
Valley Grassy Forest	10	100	17
Plains Grassy Woodland	4	15	15
Alluvial Terraces Herb-Rich Woodland	4	15	15

Figure 4.5: Location of the case study



(a) Location of the case study in the Barwon-Otway district of Victoria, Australia

(b) Map showing the distribution of the candidate treatment units within case study area in the Barwon-Otway district of Victoria, Australia

Table 4.4: Computational comparison between the five, six and seven percent treatment levels

Length of planning horizon	Solution time (seconds)		
	five percent	six percent	seven percent
5 years	22.32	13.12	11.72
10 years	462.44	38.29	17.62
15 years	4904.10	752.11	366.71
20 years	26652.91	9464.17	2384.15

Figure 4.6: Solution of Phase 2: Maps showing the location of fuel treatment and the spatial distribution of high-risk treatment units over time (in years)

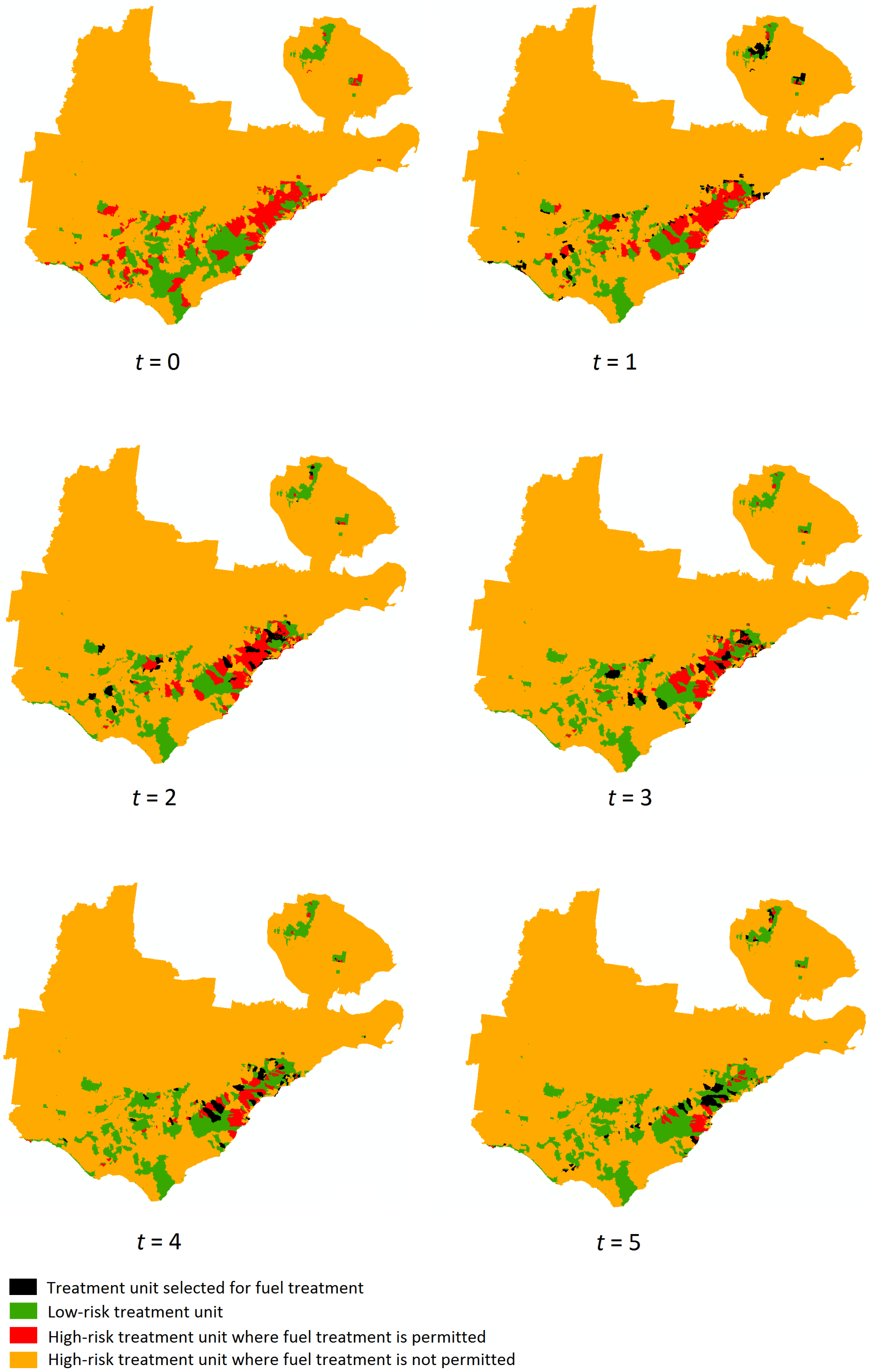
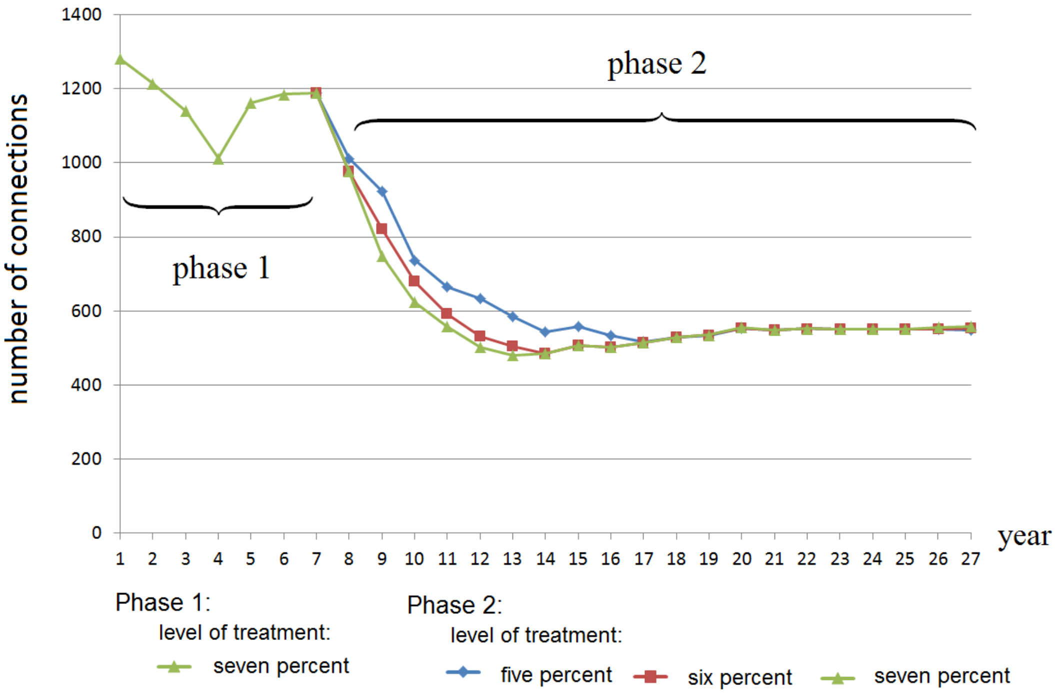


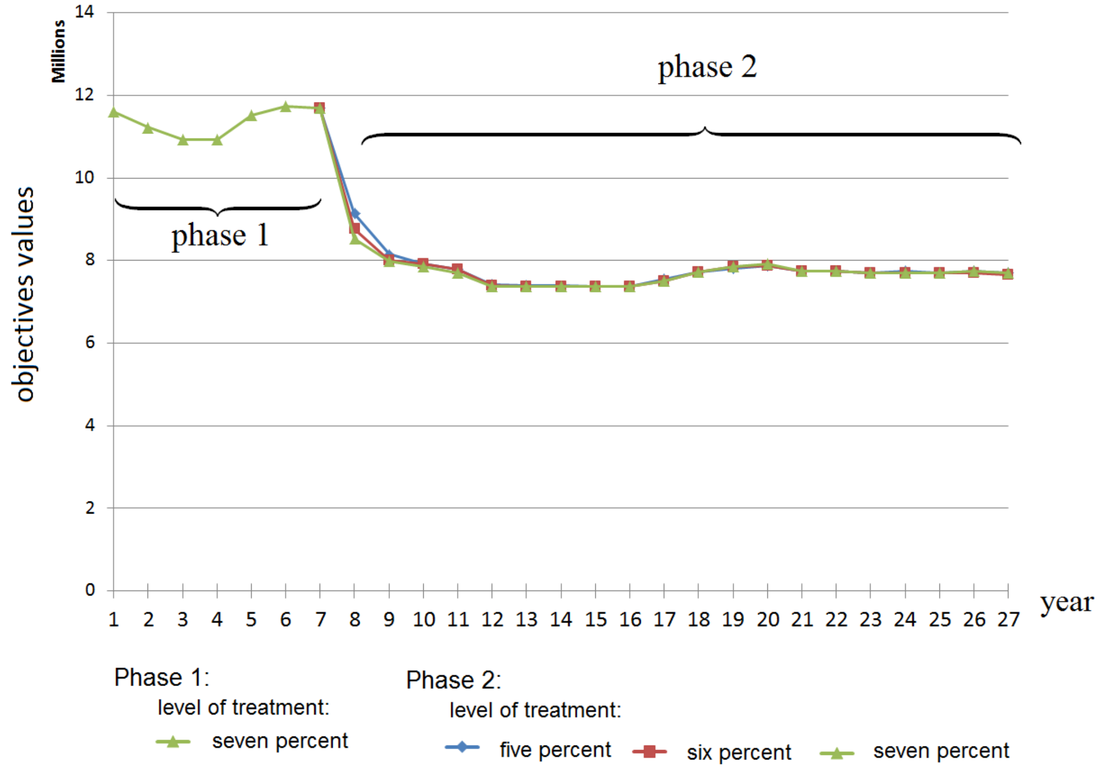
Figure 4.7: The number of connections of high-risk treatment units over time



case study. We use a real landscape with randomised data containing treatable patches, grouped into 1197 treatment units. Figure 4.6a illustrates the location of the case study in the Barwon-Otway district of Victoria, Australia. In this case study, we assume that we can only treat the public treatment units. Figure 4.6b represents the 711 candidate locations for fuel treatment. The data includes area, vegetation type and age. The minimum TFI, maximum TFI and the high-risk age threshold for each vegetation is summarised in Table 4.3. The vegetation types that do not pose any threat such as aquatic vegetation types are excluded in this chapter. Threshold values are set to their assumed values to demonstrate our approach rather than to provide an actual way of determining these values.

A set of connected treatment units is defined as a treatment unit directly

Figure 4.8: The objective function values over time



adjacent to another treatment unit, in other words, having a shared boundary. It is acknowledged it is possible for treatment units that are geographically separated to still be considered ‘connected’ as a result of the spotting behaviour of particular bark fuel types under given weather conditions. The provision of information regarding bark fuel types and prevailing weather conditions for the case study area would be a simple addition to model.

From the initial data, it was identified that 31 percent of the total treatable area in the landscape is high-risk treatable treatment units containing the patches that are over maximum TFI and no young patches. Phase 1 is run for seven percent treatment level, and would need seven years to achieve less than five percent high-risk treatment units containing old patches in the landscape. In Phase 2, we run the model presented in Section 4.3 for five, six and seven percent treatment levels. The solutions representing the high-risk area over time and the location selected for fuel treatments each year with seven percent treatment level can be seen in Figure 4.6. In this case study, we use the area of the two connected high-risk treatment units as a weight to determine the relative importance of the connectivity. However, this weight can be determined in another way, for example, by the proportion of the shared boundary between two adjacent treatment units to the perimeter of the treatment units. It can even be adjusted subjectively by the land manager if required. Figure 4.7 and 4.8 show that the connectivity of high-risk treatment units in the landscape and the objective function values decrease over time. The average of the number of connections for five, six and seven percent treatment levels are 608, 579 and 569, respectively.

The model was solved using ILOG CPLEX 12.6 with the Python 2.7 programming language using PuLP modeler. Computational experiments were performed on Trifid, a V3 Alliance high-performance computer cluster. The computational experiment used a single node with 16 cores of Intel Xeon E5-2670 64 GB of RAM. The comparison of computational time between the three different treatment levels can be seen in Table 4.4. For the ten-year planning horizon, the computational time for the three treatment levels is less than 15 minutes. For the longer planning horizon, the computational time becomes longer. The optimal solution can be obtained up to 20-year planning horizon.

4.7 Chapter summary

In this chapter, we have presented a mixed integer programming based approach to schedule fuel treatments. The model determines when, and where, to conduct the fuel treatment to reduce the fuel hazards in the landscape whilst still meeting ecological requirements. The ecological requirements considered in this chapter are the minimum and maximum Tolerable Fire Intervals (TFI) for the vegetation present. The model includes multiple vegetation types and ages in the landscape and tracks the age of vegetation in each treatment unit. To avoid deadlocks, the rules that are applied in the model are either: the treatment unit must be treated if there is an old patch in a treatment unit, or the treatment unit cannot be treated if there is a young patch in a treatment unit. In this study, spatial and temporal changes that include multiple vegetation types in a more realistic polygon-based network representation of the landscape are considered. These improve upon previous work which was limited to a single vegetation type in a regular grid, and create a more realistic approach to fuel treatment planning for land managers.

The model was illustrated in fuel treatment planning using real landscape data from the Barwon-Otway district in south-west Victoria, Australia. We ran the model for a 20-year planning horizon with five, six and seven treatment levels. The total connectivity of high-risk regions resulting from the three different treatment levels in the landscape differs substantially for the first five years and differs slightly after five years. Based on our experiments, using seven percent treatment level, the high-risk regions in the landscape can be fragmented more quickly than that of five and six percent, as expected. From the case study, the solution of this complex multi-period model can be obtained in a reasonable computational time (eight hours). In the next chapter, we enhance this study by including habitat connectivity for fauna in the landscape while fragmenting high-risk areas.

5 Fuel treatment planning maintaining habitat availability and connectivity for endangered species conservation

In chapters 3 and 4, habitat connectivity and availability within the landscape are not considered in the fuel treatment planning. In this chapter, we enhance previous work by taking into account this vital requirement. However, for simplicity, in proposing this idea, in this chapter we are dealing with a landscape comprising grid cells with a single vegetation type.

5.1 Introduction

Each year parts of the landscape are treated to reduce the overall fuel in order to diminish the risk and impact of wildfires for subsequent fire seasons. Fuel management is a cyclic activity, varying in treatment frequency, partially dictated by the vegetation community. Fuel load or biomass accumulation is a continuous ecosystem process. As such, it is infeasible to prevent all wildfires from occurring through fuel treatment. However, this activity is acknowledged as reducing the suppression efforts required with wildfires being easier to contain in areas having received fuel reduction treatments (Martell, 2015). Treating the landscape in this way breaks the connectivity of high-risk areas, helping to prevent or minimise the spread and intensity of wildfire.

Previous studies have observed the importance of landscape-level fuel treatment (Chung, 2015). The spatial arrangement of fuel treatment planning plays a substantial role in providing better protection in the landscape (Rytwinski and Crowe, 2010). Fuel arrangement can modify fire behaviour and when fragmented, can lessen the chance of large wildfires (Kim et al., 2009). The main factor that affects wildfire extent is the connectivity of 'old' untreated patches (Boer et al., 2009). Wei and Long (2014) proposed a single-period model to break the connectivity of high-risk patches by taking into account the duration and speeds of a future fire. Taking into account the vegetation dynamics over time is fundamental to accurate fuel treatment planning (Krivtsov et al., 2009). Minas et al. (2014) achieved a multi-period model for fuel treatment planning. This model breaks the connectivity of 'old' patches in the landscape to minimise fire spread and takes into account the vegetation dynamics for treated and untreated areas at each time increment. This model tracks changes in fuel for a single vegetation type, yet does not take into consideration habitat connectivity. Habitat connectivity is vital to support the ecology and genetics of local populations and endangered species (Rayfield et al., 2015). To date, no approach has been developed for the fuel treatment and habitat conservation problem.

In this chapter, we significantly extend current models by tracking and maintaining defined levels of habitat connectivity over time, in addition to reducing and fragmenting high-risk areas across the landscape. The model we present is the first multi-period fuel treatment model that takes into account the habitat connectivity to be modelled and solved using exact optimisation. In this proposed approach, the minimum and maximum Tolerable Fire Intervals (TFIs) are used to describe the ecological fire requirements of the ecosystem. We assume that fuel treatment can support the ecosystem's health if it is conducted when the vegetation age is between these two intervals. The vegetation types whose age are over the maximum TFI are also treated to help maintain vegetation condition and renewal.

The efficacy of the applications of fuel treatment remains debated among experts according to different perspectives (Penman et al., 2011). Fuel treatments reduce the overall fuel load in landscapes (Martell, 2015) that at the same time may result in significant habitat modification for species populations living within the treated area. If habitat availability in the landscape is not maintained, populations may be adversely affected, leading to local extinctions where minimum viable population thresholds are no longer met. For example, the Mallee emu-wren, a native bird of Australia, depends on 15-year-old mallee-Triodia vegetation (Brown et al., 2009) for survival. This vegetation recovers very slowly after fuel treatments, and the Mallee emu-wren is unable to survive in vegetation aged less than 15 years. Similarly, frequent fires in California can destroy the mature coastal sage scrub habitat required for the coastal cactus wren and the California gnatcatcher on which these species rely (Conlisk et al., 2015). If we want to conserve these species, it is important to maintain the availability and connectivity of their habitats. The question that then arises is: Can fuel treatments be scheduled to break the connectivity of high-risk areas while limiting its negative impacts on the ecosystem?

Similarities exist between the fuel treatment problem described here and the forest harvesting problem and its impact on the environment. Both of these problems consider vegetation dynamics and can be seen as a 'timing problem', meaning that the risk and values change over time as the vegetation grows. In the fuel treatment problem, an area is treated to reduce fuel load; in the forest harvesting problem, an area is harvested using mechanical clearing for timber production. Both activities have adverse side effects to natural ecosystems, such as habitat loss. Previous studies in the forest harvesting problem have taken into account some ecological requirements. Öhman and Wikström (2008) proposed an exact method for long-term forest planning to maintain the biodiversity of the forest. They believe that biodiversity in the forest ecosystem can be maintained by minimising the total perimeter of old forest patches so that the fragmentation of old forest is reduced. Hence, the compactness of the habitat for species can be achieved. The model was run in a five-yearly planning horizon across a landscape that comprised 924 stands within a reasonable computational time. However, their model did not consider habitat connectivity across time. Addressing this shortcoming, Könnyű et al. (2014) proposed a model that ensures mature forest patches are temporarily connected between time-steps while scheduling forest harvesting. The model works well and does not substantially reduce timber revenues. However, like the previous fuel treatment models, this model does not take into account the overall habitat connectivity of each period, nor does it track the habitat connectivity across the entire planning horizon, both of which are important for the persistence of species.

Therefore, this chapter brings together ideas from previous research in forest harvesting operations by incorporating habitat connectivity across time as well as maintaining habitat connectivity within each time period, to formulate an approach to the fuel treatment and habitat planning problem. Furthermore, the fuel treatment problem requires the landscape to be fragmented, whereas in contrast, the forest management problem seeks to maintain clusters in the landscape.

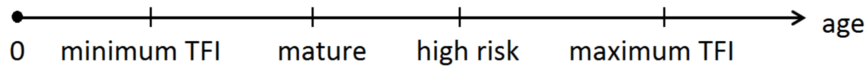
A Mixed Integer Programming (MIP) approach is presented here for fuel treatment planning to fragment the high-risk areas as much as possible while still considering the TFIs to support biodiversity and maintaining two forms of habitat connectivity in the landscape with a single vegetation type and a single species. We assume that the animal species can relocate to a neighbouring area that has similar habitat characteristics (for example, the same vegetation age stage). In the first form of habitat connectivity, each treated region forming a habitat has to be connected to an alternative habitat for the species to relocate (that is, neighbouring mature areas). Second, at any period, a minimum acceptable target is set for habitat connectivity to conserve species. The model is then demonstrated on a series of hypothetical landscapes comprising rectangular grid cells.

5.2 Model formulation

In this chapter, cells represent the candidate locations for fuel treatment in a landscape. For each cell, time since treatment or fuel age (years) is tracked. Fuel treatment determines the cell's fuel age at each period. The cell's fuel age is reset to zero if the cell is treated or incremented by one if untreated. Each cell has its minimum and maximum tolerable fire intervals (TFIs). Each cell also has its mature age threshold which determines the suitability of the cell as a habitat and high-risk age threshold which determines whether the cell poses a high level of risk for fire. We assume that these two thresholds are between the minimum and the maximum TFIs. The mature age threshold is less than the high-risk age threshold. The relationship between these thresholds are represented in Figure 5.1. If ignition occurs , the fire will potentially spread through connected high-risk cells. Therefore, if these high-risk cells are disconnected (using fuel treatment), the chance of disastrous wildfires should be reduced. At the same time, this fuel treatment activity destroys the habitat for species. For the purposes of illustration, we assume that the animal species of our concern live in mature or older cells. To conserve this species, for each habitat cell to be treated, we have to provide an alternative habitat (i.e. neighbouring mature cells) for the species to occupy during the next period.

The following mixed integer programming model is formulated to optimally

Figure 5.1: The relationship between the minimum TFI, mature, high risk, and the maximum TFI threshold values



decide which cells should be treated each year to break the connectivity of high-risk cells in the landscape while providing continuing of habitat for the species of concern.

Sets:

C is the set of all cells in the landscape

Φ_i is the set of cells connected to cell i

T is the planning horizon

Indices:

i = cell

t = period, $t = 0, 1, 2, \ldots T$

Parameters:

a_i = initial fuel age of cell i

ρ = treatment level (in percentage), i.e. the maximum proportion of the total area in a landscape selected for treatment

R = the total area of cells in the landscape

c_i = area of cell i

d_i = high-risk age threshold for cell i

m_i = mature age threshold for cell i

G_t = desired target of mature cell connectivity in time t

$MaxTFI_i$ = maximum tolerable fire interval (TFI) of cell i

$MinTFI_i$ = minimum TFI of cell i

Decision variables:

$A_{i,t}$ = fuel age of cell i in time t

$$x_{i,t} = \begin{cases} 1 & \text{if cell } i \text{ is treated in time } t \\ 0 & \text{otherwise} \end{cases}$$

$$Mature_{i,t} = \begin{cases} 1 & \text{if cell } i \text{ is classified as `mature cell' in time } t \\ 0 & \text{otherwise} \end{cases}$$

$$HabitatConn_{i,j,t} = \begin{cases} 1 & \text{if connected cell } i \text{ and } j \text{ are both mature cell cells in time } t \\ 0 & \text{otherwise} \end{cases}$$

$$High_{i,t} = \begin{cases} 1 & \text{if cell } i \text{ is classified as high-risk cell in time } t \\ 0 & \text{otherwise} \end{cases}$$

$$HighConn_{i,j,t} = \begin{cases} 1 & \text{if connected cell } i \text{ and } j \text{ are both high-risk cells in time } t \\ 0 & \text{otherwise} \end{cases}$$

$$Old_{i,t} = \begin{cases} 1 & \text{if cell } i \text{ is classified as `over-the-maximum-TFI'} \\ & \text{cell in time } t \\ 0 & \text{otherwise} \end{cases}$$

minimise the connectivity of high-risk cells

$$z = \sum_{t=1}^{T} \sum_{i \in C} \sum_{j \in \Phi_i, i<j} HighConn_{i,j,t} \tag{5.1}$$

subject to

$$\sum_{i} c_i x_{i,t} \leq \rho R,\ t = 1 \ldots T, \forall i \in C \tag{5.2}$$

$$A_{i,0} = a_i,\ \forall i \in C \tag{5.3}$$

$$A_{i,t} \geq A_{i,t-1} + 1 - M_1 x_{i,t},\ t = 1 \ldots T, \forall i \in C \tag{5.4}$$

$$A_{i,t} \leq M_2(1 - x_{i,t}),\ t = 1 \ldots T, \forall i \in C \tag{5.5}$$

$$A_{i,t} \leq A_{i,t-1} + 1,\ t = 1 \ldots T, \forall i \in C \tag{5.6}$$

$$A_{i,t} - d_i \leq M_3 High_{i,t} - 1,\ t = 1 \ldots T, \forall i \in C \tag{5.7}$$

$$A_{i,t} \geq d_i High_{i,t},\ t = 1 \ldots T, \forall i \in C \tag{5.8}$$

$$High_{i,t} + High_{j,t} - HighConn_{i,j,t} \leq 1,\ t = 1 \ldots T, \forall j \in \mathit{\Phi}_i, i < j, \forall i \in C \tag{5.9}$$

$$A_{i,t} - m_i \leq M_4 Mature_{i,t} - 1,\ t = 1 \ldots T, \forall i \in C \tag{5.10}$$

$$A_{i,t} \geq m_i Mature_{i,t},\ t = 1 \ldots T, \forall i \in C \tag{5.11}$$

$$\sum_{j \in \mathit{\Phi}_i} Mature_{j,t} \geq x_{i,t},\ t = 1 \ldots T, \forall i \in C \tag{5.12}$$

$$Mature_{i,t} + Mature_{j,t} - HabitatConn_{i,j,t} \leq 1,\ t = 1 \ldots T, \forall j \in \mathit{\Phi}_i, i < j, \forall i \in C \tag{5.13}$$

$$Mature_{i,t} + Mature_{j,t} \geq 2HabitatConn_{i,j,t},\ t = 1 \ldots T, \forall j \in \mathit{\Phi}_i, i < j, \forall i \in C \tag{5.14}$$

$$\sum_{i \in C} \sum_{j \in \mathit{\Phi}_i, i < j} HabitatConn_{i,j,t} \geq G_t,\ t = 1 \ldots T, \forall i \in C \tag{5.15}$$

$$A_{i,t} - MaxTFI_i \leq M_5 Old_{i,t} - 1,\ t = 0 \ldots T-1, \forall i \in C \tag{5.16}$$

$$A_{i,t} \geq MaxTFI_i Old_{i,t},\ t = 0 \ldots T-1, \forall i \in C \tag{5.17}$$

$$Old_{i,t-1} + \frac{1}{\mid \Phi_i \mid} \sum_{j \in \Phi_i} Mature_{j,t} \leq 1 + x_{i,t},\ t = 1 \ldots T, \forall i \in C \tag{5.18}$$

$$A_{i,t-1} \geq MinTFI_i x_{i,t},\ t = 1 \ldots T, \forall i \in C \tag{5.19}$$

$$x_{i,t},\ High_{i,t},\ HighConn_{i,j,t},\ Mature_{i,t},\ Old_{i,t} \in \{0,1\} \tag{5.20}$$

The objective function (5.1) minimises the connectivity of high-risk cells in a landscape across planning horizon. Constraint (5.2) specifies that the total area selected for fuel treatment should not exceed the total area allocated for fuel treatment in each period

Constraint (5.3) sets the initial fuel age in a cell. Constraints (5.4) to (5.6) track the fuel age of each cell. Constraints (5.4) and (5.5) indicate that if a cell is not treated, then the cell's fuel age will be incremented by one in the following period. Whereas if a cell is treated, the fuel age will reset to zero. Constraints (5.4) and (5.6) increment fuel age by exactly one year if the cell is not treated.

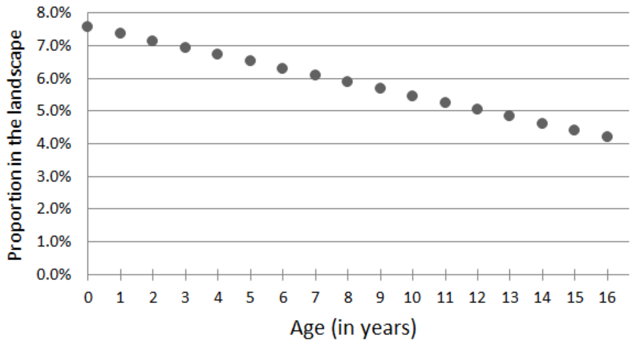
Constraints (5.7) and (5.8) use binary variable $High_{i,t}$ to classify a cell to be a high-risk cell if and only if the fuel age exceeds a threshold value. In Constraint (4.10), $HighConn_{i,j,t}$ takes the value one if connected cells i and j are both classified as high-risk cells in time t.

Constraints (5.10) to (5.11) classify a cell to be a 'mature' cell, if and only if the fuel age is over the mature age threshold. Constraint (5.12) states that we cannot treat a cell this period unless there is at least one neighbouring mature cell in the following period.

In this model, we also consider maintaining habitat (mature-cell) connectivity in the landscape for each period. Constraints (5.13) and Constraint (5.14) ensure that $HabitatConn_{i,j,t}$ takes exactly value one if and only if connected cells i and j are both classified as mature cells at time t. Constraint (5.15) makes sure that the number of habitat connections each year is greater than the desired target, G_t.

Constraints (5.16) to (5.17) classify a cell to be an 'over- the-maximum-TFI' cell (if and only if the fuel age is over the maximum TFI). Constraint (5.18) ensures that a cell must be treated if the cell's fuel age is over maximum TFI, and there is at least one neighbouring mature cell in the following period. This constraint avoids a deadlock that may occur when the cell's fuel age is over the maximum TFI and there are no neighbouring mature cells for the next period. In this study, we break the deadlock in favour of mature cell availability. Constraint (5.19) ensures that the cell with fuel age less than the minimum TFI cannot be treated. Constraint (4.17) ensures that the decision variables take binary values.

Figure 5.2: Proportion of initial cell's fuel age in the landscape for the computational experiments



5.3 Model illustration

In this section, we demonstrate the approach discussed in Section 4.3 using hypothetical random landscapes comprising 100 grid cells, generated using the NLMpy package (Etherington et al., 2015). We assume that there is a single fuel type in the landscape, with the thresholds of mature and high-risk ages set as 8 and 12 years old, respectively. The minimum and the maximum TFIs are chosen as 2 and 16 years, respectively. The initial fuel ages in the landscape are between 0 and 16 years, this means that not all the cells are categorised as high risk. Figure 5.2 represents the assumed distribution of the initial cell fuel age. A cell is assumed to be connected to its immediate neighbouring cells that have shared boundaries (Figure 5.3). Suppose that there are at most ten cells to be treated each year (ten percent of the total area in the landscape), and the length of planning horizon is 13 years.

Initially, the landscape has 13 high-risk cell connections and 39 habitat connections that we want to maintain over the planning horizon, as illustrated in Figure 5.4. In this model illustration, we compare four different settings (Table

Figure 5.3: The definition of connected cells. Cell 5 is considered connected to cells 6 (right), 4 (left), 2 (up) and 8 (down)

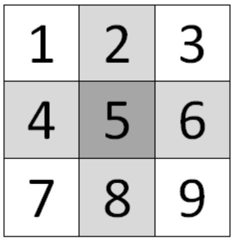


Figure 5.4: Illustration of initial high-risk cell and habitat connectivity in the landscape, the arrow (↔) represents one connection

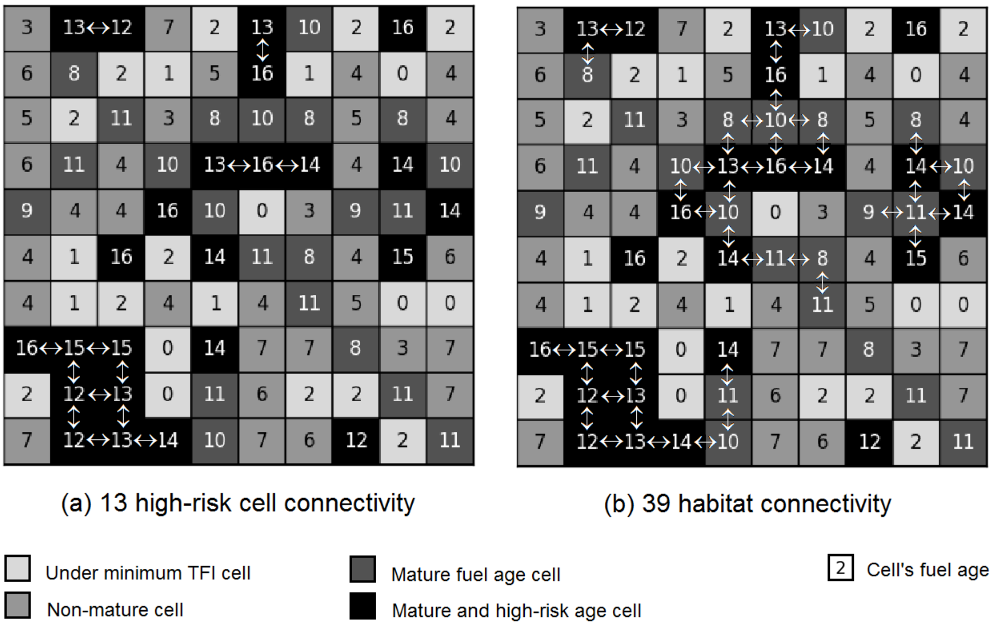


Table 5.1: Four settings for the model illustration and the computational experiments

	Limit on overall habitat connectivity	Neighbouring habitat cell requirement for treatment
Setting 1	G_t is set to the initial number of habitat connectivity of the landscape	yes
Setting 2	G_t is set to the initial number of habitat connectivity of the landscape	no
Setting 3	G_t is set to zero	yes
Setting 4	G_t is set to zero	no

5.1). In the first and second settings, we maintain the initial number of habitat connectivity, at a minimum level of 39 connections. In the first setting we enforce the requirement that a cell can only be treated if there is a neighbouring cell forming a suitable habitat, but in the second setting we do not have that requirement enforced. In the third setting, the neighbouring habitat cell requirement is enforced without maintaining the overall habitat connectivity. Setting 4 represents the base case with the only aim of fragmenting high-risk cells without any species conservation consideration. All settings are different in terms of the first and second form of habitat connectivity. However, they are the same in terms of the requirement for conducting the fuel treatment planning between the minimum and the maximum TFIs. The python codes used to run the experiments in this chapter are presented in the Appendix.

The solutions to settings 1 to 4 are illustrated in Figure 5.5, Figure 5.6, Figure 5.7 and Figure 5.8, respectively. Under settings 1, 2 and 3, in some years, the number of treated cells is less than the treatment level (ten percent of the total

Figure 5.5: Fuel treatment schedule with ten percent treatment level and thirteen-year planning horizon for the first setting, G_t= initial

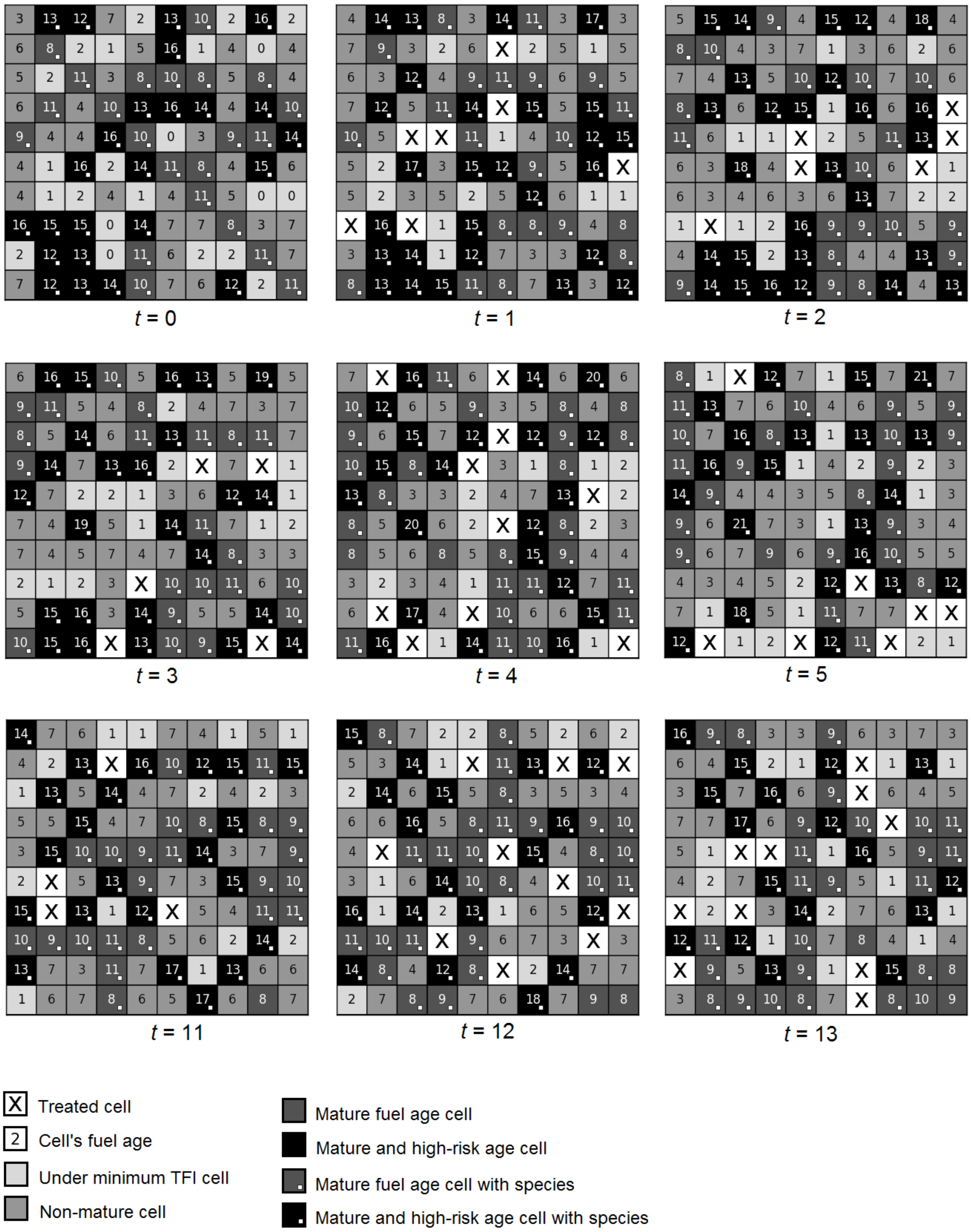


Figure 5.6: Fuel treatment schedule with ten percent treatment level and thirteen-year planning horizon for the second setting, G_t= initial, without applying Constraint (12)

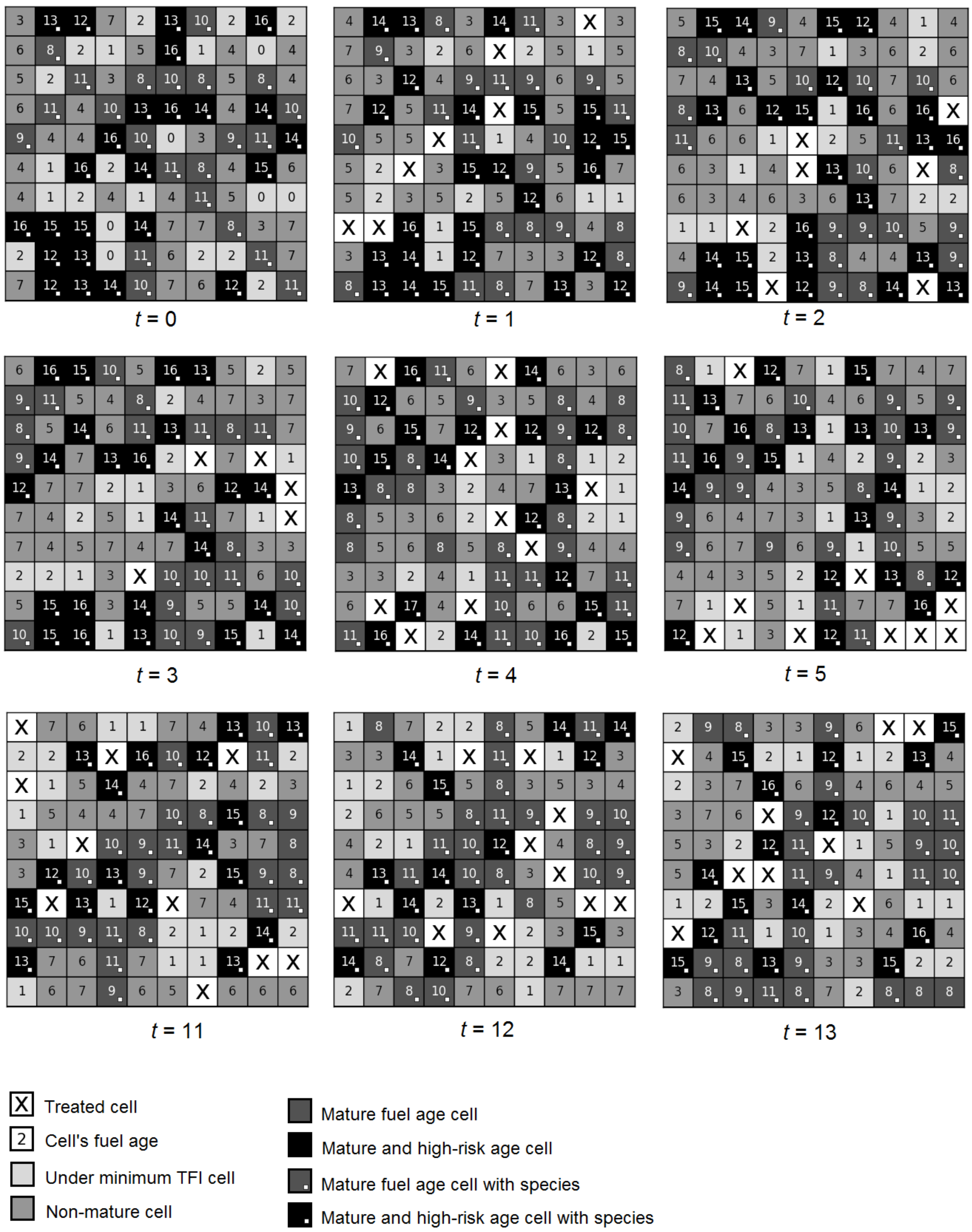


Figure 5.7: Fuel treatment schedule with ten percent treatment level and thirteen-year planning horizon for the third setting, $G_t = 0$

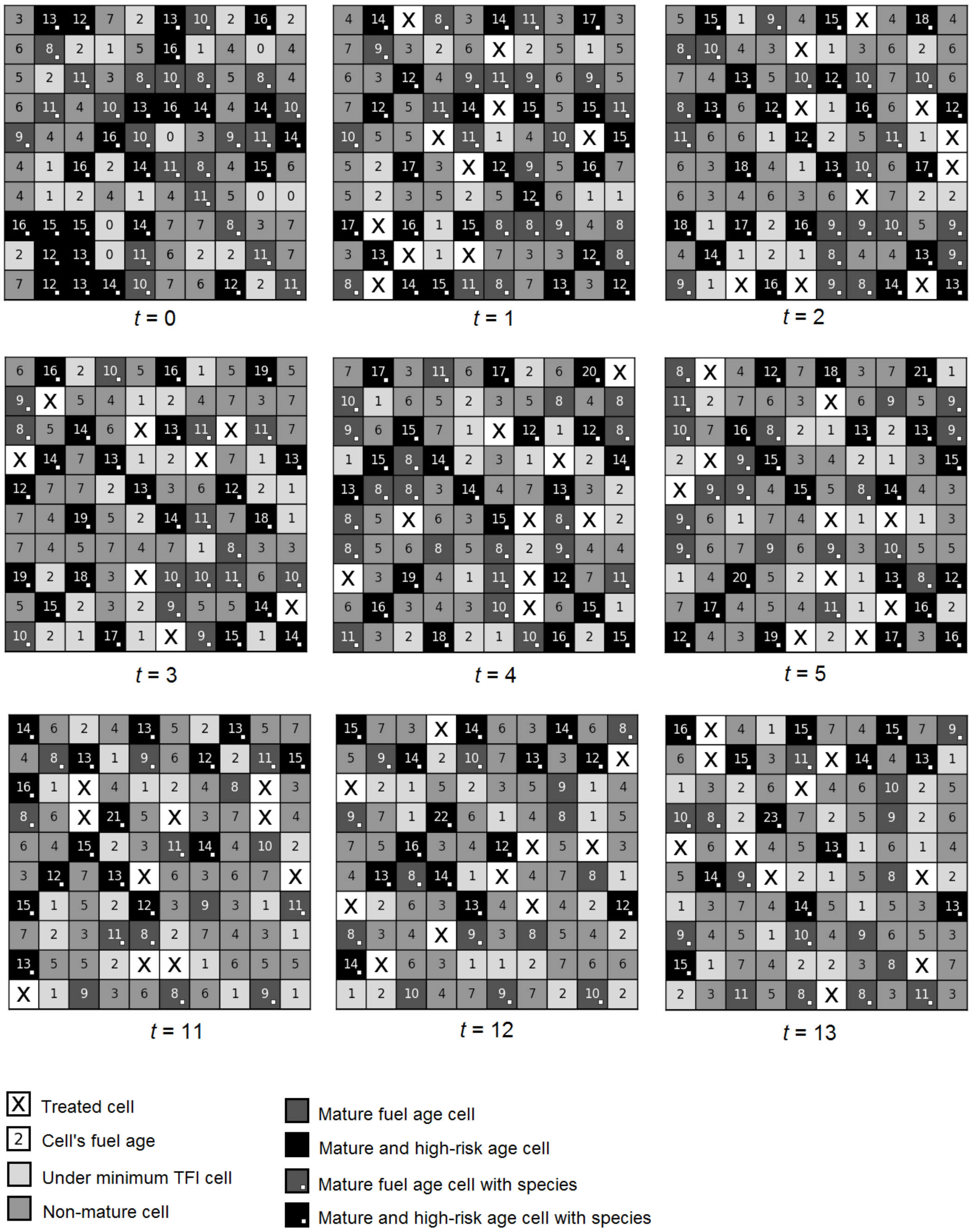


Figure 5.8: Fuel treatment schedule with ten percent treatment level and thirteen-year planning horizon for the fourth setting, $G_t = 0$, and without applying Constraint (12)

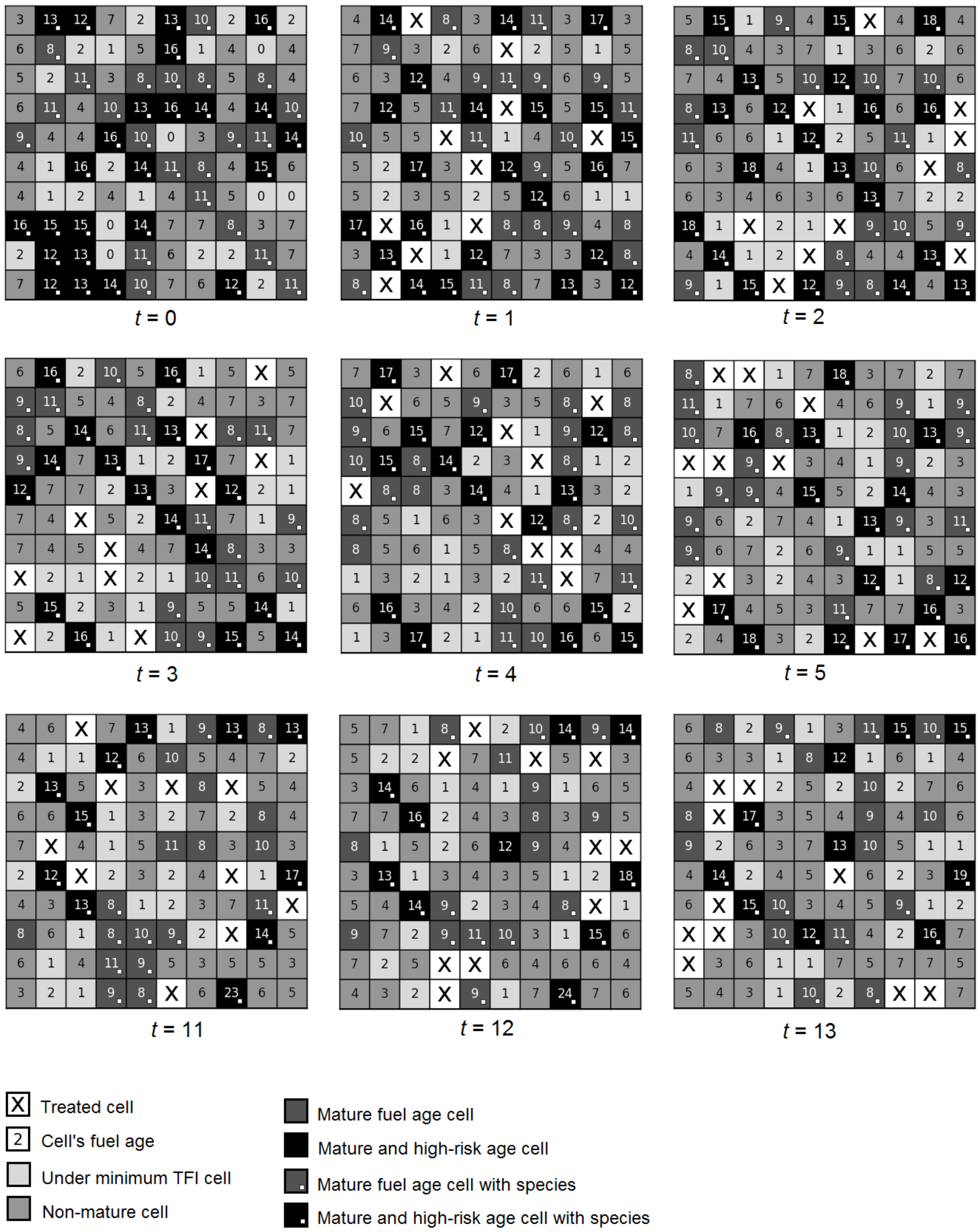
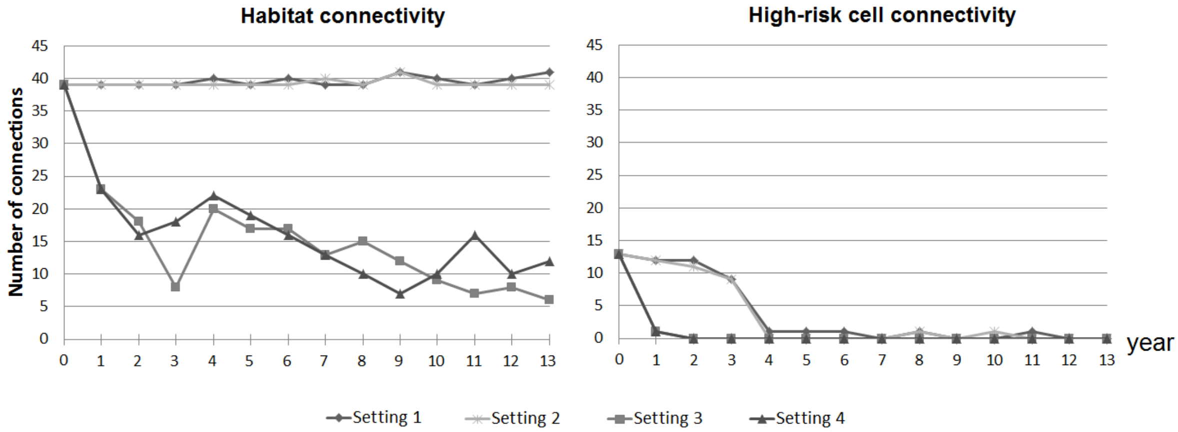


Figure 5.9: The number of habitat connections and high-risk cells connections of the model illustration (10 × 10 grid cells, ten-percent treatment level)



landscape) due to the habitat conservation requirements. In settings 3 and 4, the high-risk cells in the landscape are fully fragmented more quickly than settings 1 and 2 within the planning horizon, because under settings 1 and 2, there are 39 connections of the habitat that need to be maintained each year. All settings show that while this landscape is homogeneous in terms of fuel type, there is no particular pattern in treating the cells each year. This irregular pattern is due partially to factors such as the varied initial ages and minimum and maximum TFI requirements. Although the cells treated in the first year would grow over the high-risk threshold by year 13, the figures show that the cells selected for fuel treatment in year one may or may not be re-selected in year 13. Results of settings 1 and 3 show that in the absence of adjacent mature habitat cells, the cells exceeding the maximum TFI cannot be treated. The number of habitat connectivity and the high-risk area connectivity resulting from the four settings of the model illustration are represented in Figure 5.9.

Figure 5.10: The proportions of high-risk cells and mature cells in the landscape for the model illustration (10 × 10 grid cells, ten-percent treatment level)

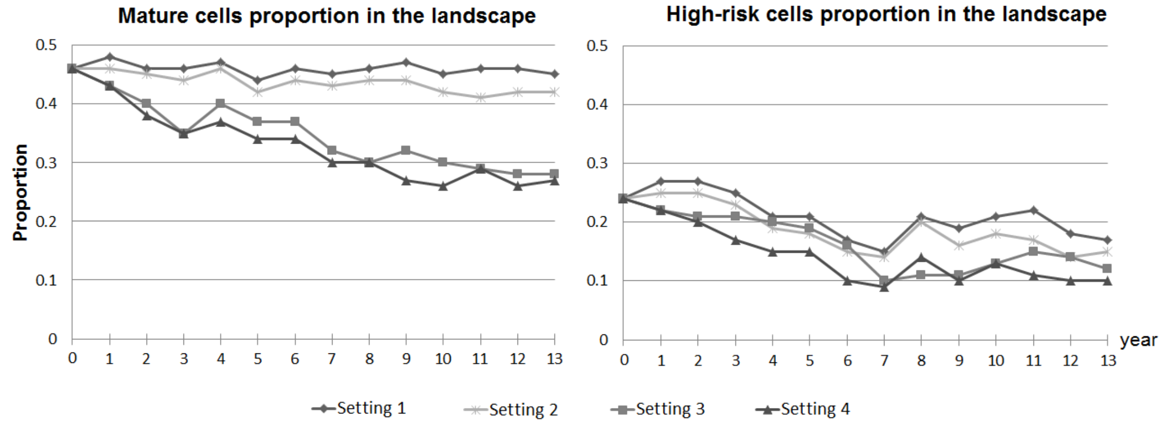
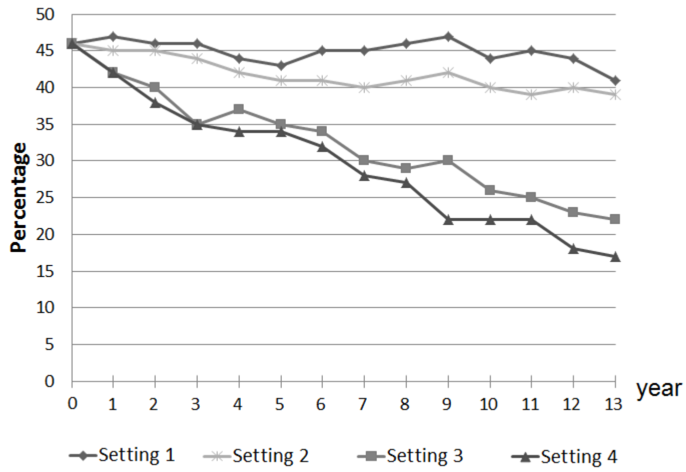


Figure 5.11: Proportion of mature cells with species in the landscape for the model illustration (10 × 10 grid cells)



From the solutions of this model illustration, we also track the existence of animals in mature cells in the landscape over the planning horizon for the four settings, as represented in figures 5.5 to 5.8. We assume that initially, all mature/high-risk cells are populated by an endangered species. The species can only move from one habitat to another habitat, and they will not populate a new habitat unless there is a direct connection. Here, we use the same definition of connection as illustrated in Figure 5.3, meaning that in a single period, the species can only move one cell away to four neighbouring cells (right, left, up, or down). The proportion of mature cells with species in the landscape for the model illustration is represented in Figure 5.11.

5.4 Computational experiments

A series of computational experiments were conducted by using ILOG CPLEX 12.6.2 with the Python 2.7.2 programming language using PuLP modeller. The experiments were ran on Trifid, a computer cluster of V3 Alliance. A single node with 16 cores of Intel Xeon E5-2670 64 GB of RAM is used for the computational experiments. The landscape sizes are 10×10 and 15×15 grid cells.

The computational experiments are conducted with these following steps. Firstly, for each landscape size, 30 hypothetical landscapes are generated using NLMpy package. Figure 5.2 represents the assumed percentages of the initial cell's fuel age in each landscape. Then, we ran four different settings based on Table 5.1. In the first two settings, we evaluated the initial number of connected habitat (connected mature cells) for each landscape. Based on this result, we maintain this number over the planning horizon.

For each setting, we ran computational experiments for 30 landscapes for each landscape size (10×10 and 15×15 grid cells), with ten-percent treatment level and a ten-year planning horizon. In the first and second settings, we found that for some landscapes, it is impossible to maintain the initial number of habitat over the planning horizon. To deal with this infeasibility, we ran the model by assigning a lower value of G_t for the first years in a planning horizon, and setting the higher value (the initial number of habitat connectivity) of G_t for the rest of the year within the planning horizon only once it is feasible.

The 95% confidence intervals of the number of high-risk cell connectivity and habitat connectivity for the four settings are summarised in Figure 5.12. This figure shows that for the first two settings the number of high-risk cells connectivity decreases over time, and the number of habitat connectivity is relatively stable and can be maintained at their initial level. For the third and fourth settings, the number of high-risk cells connectivity reaches zero since year two, but the number of habitat connectivity decreases significantly over time. Figure 5.13 summarises the 95% confidence intervals of the proportions of the high-risk cells and mature cells for the four settings. The third and fourth settings provide less high-risk cells or mature cells than the first and second settings, and the first setting outperforms the others.

The proportion of mature cells with species in the landscape for these computational experiments is summarised in Figure 5.14. The difference between

Figure 5.12: 95% confidence interval of high-risk cell connectivity and habitat connectivity for the computational experiments

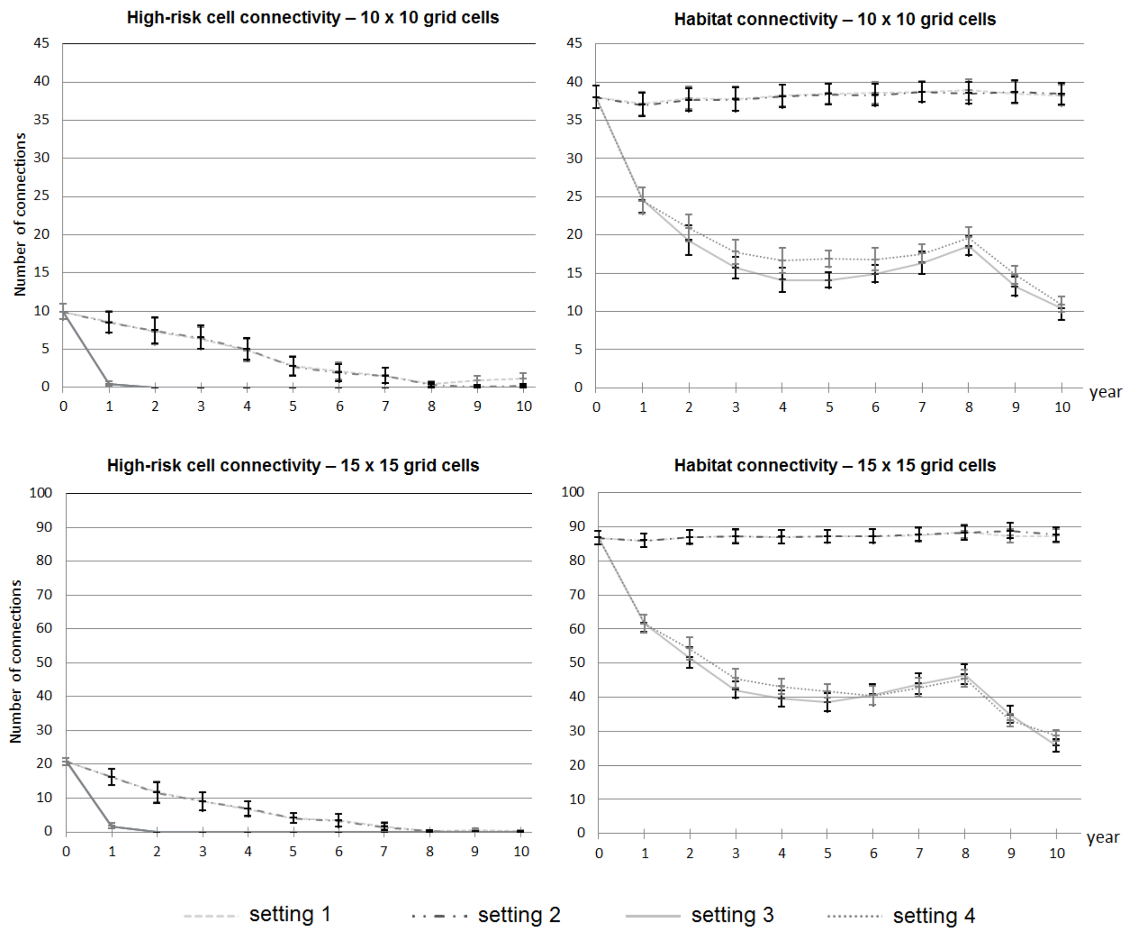


Figure 5.13: 95% confidence interval of proportions of high-risk cells and mature cells in the landscape for the computational experiments

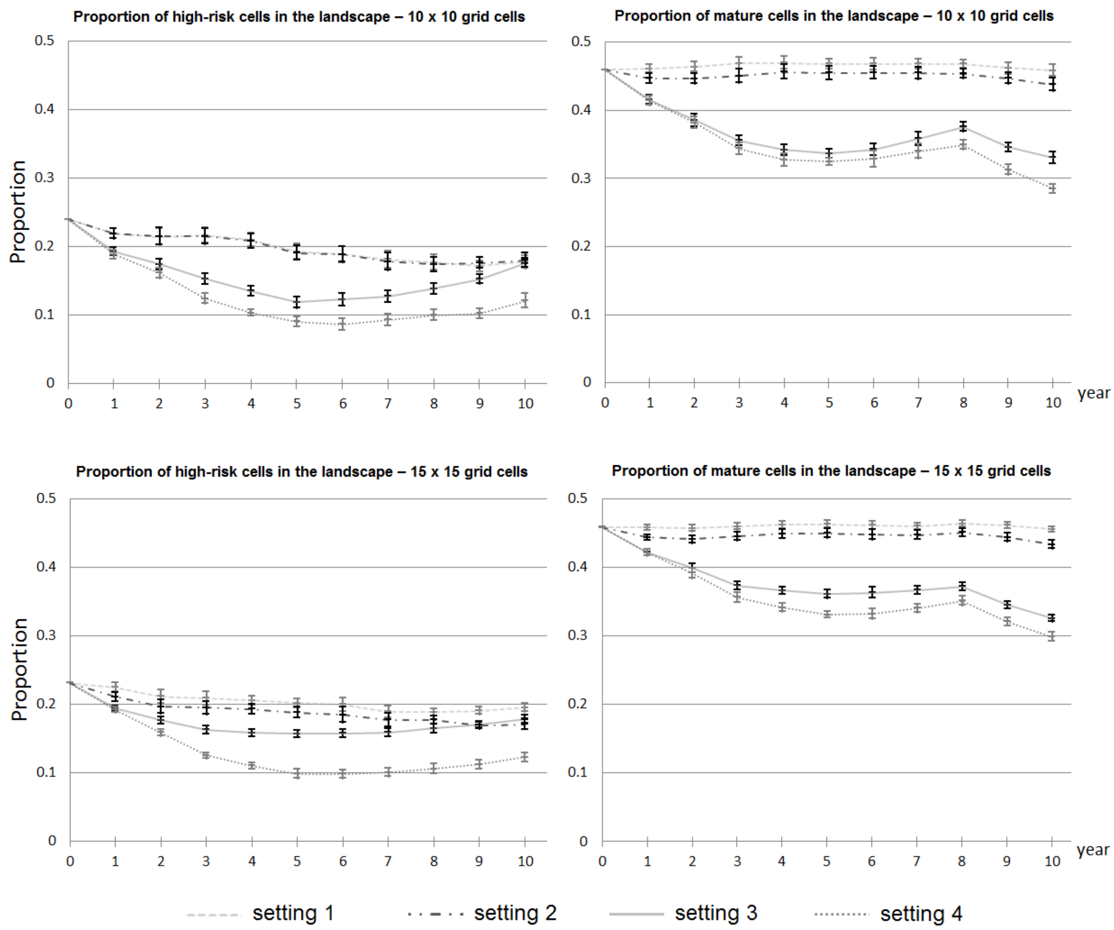
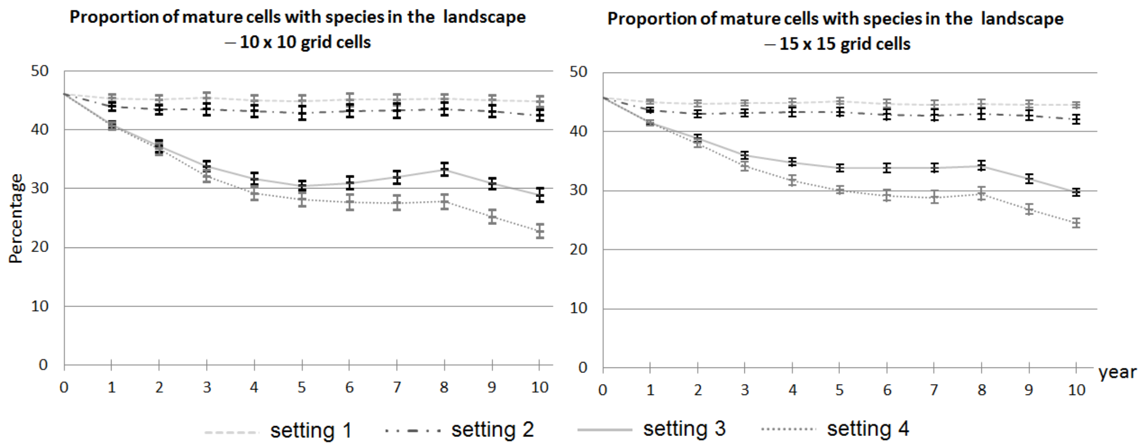


Figure 5.14: Proportion of mature cells with species in the landscape for the computational experiments



settings 3 and 4 clearly shows that requirement of having a neighbouring mature cell for treatment itself is important in the absence of maintaining the overall habitat connectivity. The difference between the first two settings and the last two settings shows that the overall limit on connectivity works well for this measure.

5.5 Chapter summary

In this chapter, we proposed a mixed integer programming approach to schedule fuel treatments. This approach tracks the age of vegetation in each cell in each year and optimally decides when and where to conduct fuel treatment to fragment high-risk cells in the landscape while meeting ecological requirements. Three types of ecological constraints were considered comprising the following; the minimum and maximum Tolerable Fire Intervals (TFIs); the availability of suitable habitat adjacent to areas being treated at any time; maintain the initial level of habitat connectivity in the landscape throughout the planning horizon. The application of the model to hypothetical landscapes demonstrated that the objective could be achieved while meeting the ecological constraints discussed above.

The problem dealt with in this chapter has similarities to the problem of scheduling forest harvests. An important difference in our work is that after treatment (such as controlled burning) we require high-risk areas to be as dispersed as possible to reduce the risk of wildfires spreading over large areas. This is not a consideration in the forest harvesting problem. Our problem is further complicated by the two separate requirements of ensuring sufficient contiguity of habitat at any particular time and the need for appropriate habitat to neighbour any area that is to be treated. These two requirements together also need to ensure that fauna do not become trapped in an area that needs treatment.

We hope that for practical purposes, our approach can assist fire and land management agencies in making their decisions about timing and locations of future fuel treatments while considering critical ecological requirements.

6 Conclusion

This chapter provides a summary of the research conducted in this thesis, followed by an appraisal of the contributions and recommendations for future work.

6.1 Thesis summary

In Chapter 1, a general view of the research problem in fuel management was presented. We identified that the management involves long-term planning of fuel treatment activities and includes many factors and complicated spatio-temporal relationships in mitigating the negative impact of wildfires. The limitation of many existing models is that the models take into account a single-period or deal with a single vegetation type. Furthermore, fuel treatment planning should integrate the ecological requirements of the ecosystem. This provided the motivation for this study. The research objectives and scope were outlined, followed by an overview of the structure of this thesis.

In Chapter 2, we presented a literature review on the development of fuel management over recent decades, related to Objective 1 of this thesis. We categorised the modelling efforts for the fuel treatment planning as simulation/heuristic optimisation, multi-objective optimisation, dynamic programming and mixed integer programming. We found that MIP approaches provide a lot of opportunities to address long-term strategies for fuel reduction planning.

In Chapter 3, in fulfilment of Objective 2 of this thesis, we formulated a long-term fuel reduction planning model. The model takes into account multiple vegetation types with differing non-linear fuel accumulation functions. The objective of this model is to minimise fuel load over the planning horizon. We compared the exact MIP approach and heuristic approaches. A model demonstration was conducted using randomised data of 711 treatment units in the Barwon-Otway district of Victoria.

Addressing Objective 3 of this thesis, in Chapter 4 we formulated a MIP model to reduce the spatial connectivity of high-risk regions in a landscape comprising multiple vegetation types. This multi-period model takes into account the vegetation growth each year. The model was applied using randomised landscape data comprising 1197 treatment units from the Barwon-Otway district in south-west Victoria, Australia. The candidate locations for the fuel treatment planning were 711 public treatment units.

In Chapter 5, related to Objective 4 of this thesis, we designed a MIP model to disconnect high-risk regions while maintaining the habitat connectivity. The model ensures that at the time an area is treated, a suitable neighbouring habitat is available to allow fauna to relocate. The efficacy of the model was demonstrated and evaluated in a series of computational experiments with a hypothetical landscape. The computer programmes used to run the experiments in this chapter are displayed in the Appendices.

6.2 Contributions of this thesis

The work in this thesis represents a development process where later work builds on the earlier work. The main contributions in regards to the advancement of fuel treatment models are listed below.

Contribution 1 *The formulation of new multi-period models applicable to a real landscape comprising multiple vegetation types*

The main contribution of this thesis is that the models can be applied to a real landscape of multiple vegetation types and ages. In real landscapes, a treatment unit may comprise a large number of polygons, and there can be more than one polygon within the same treatment unit that has the same vegetation type but different ages. Since it is computationally prohibitive to take into account multiple ages of the same vegetation in a treatment unit, raw data is preprocessed so that all vegetation of a given type within a treatment unit will be of the same age. The models are capable of keeping track of the vegetation age for each treatment unit in a landscape. The objective function of the first model is to reduce the total fuel load in a landscape with differing non-linear fuel accumulation function.

Contribution 2 *The design of a new multi-period model to disconnect high-risk* *areas for a landscape comprising multiple vegetation types*

The second model in this thesis significantly improves upon many previous existing studies which were limited to a single vegetation type in homogeneous grid cells. The model can handle multiple vegetation types implemented within a polygon-based network. Thus, a more realistic approach to fuel treatment planning can be achieved. Because a 'high-risk' age threshold is known for each vegetation type, whether a treatment unit is a high-risk treatment unit or not at any given time can be determined. From this, the optimal strategy to fragment high-risk areas can be achieved.

Contribution 3 *The development of an integrated multi-period model that takes into consideration the aim of both disconnecting high fire risk areas and the availability of connected habitat for fauna*

The third model in this thesis is capable of maintaining habitat connectivity while reducing the risk of wildfires. The approach ensures that at the time an area is treated a suitable neighbouring area is available to allow fauna to relocate. This is important to reduce the negative impact of fuel treatment practices.

All of the models have been demonstrated using randomised real data, and randomly generated hypothetical data. Our experiments show that the approaches work well, and it is possible to perform fuel treatment and meet critical ecological requirements. The third model was demonstrated in a landscape comprising grid cells. However, the formulation of the model is also valid for real landscapes. The models determine the optimal locations and timing to conduct the fuel treatments to reduce the risk of wildfires in the landscape. The ecological requirements were described by the minimum and maximum Tolerable Fire Intervals (TFIs) for the vegetation types present. The models take into account these vital requirements in formulating a multi-period landscape-level fuel treatment planning.

6.3 Recommendations for future research

In this section, we provided some opportunities to extend this work.

Proposal 1 *The development of integrated multi-period models in fire management*

Our approach can be extended by integrating fuel management into the broader context of fire management, such as incorporating fuel management with locations of fire suppression resources.

Proposal 2 *The inclusion of weather and topography factors*

Among the three factors that affect fire behaviour (fuel load, weather and topography), fuel load was the main factor that we considered. Due to the weighting coefficients in the objective function of our models, our approach can potentially handle other factors.

Proposal 3 *The incorporation of multiple vegetation types, and multiple species of fauna in the model of Chapter 5*

The first two models in this thesis have included multiple vegetation types and ages in the landscape. The third model (Chapter 5), however, was limited to a single vegetation type and a single fauna. It can be extended by incorporating multiple vegetation types and multiple fauna.

Proposal 4 *The development of multi-objective optimisation approaches for the model of Chapter 5*

The availability and connectivity of habitat in Chapter 5 are specified within the constraint instead of within an objective function. For future research, our model can be used to perform multi-objective optimisation so that the trade-offs between performance objectives such as reduction in risk of wildfires and habitat needs can be analysed.

A disincentive to make full use of the capabilities of the models developed is the time required for computation. Improvements in computing power and optimisation algorithms will make it increasingly possible to use these models to gain further insight into complex fuel management problems.

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Appendix

The python codes needed to conduct the simulations for Chapter 5

```
# -----------------------------------------------------------------------
# Landscape.py
# -----------------------------------------------------------------------
# This script is intended to generate initial cell fuel age
# The output of this script will be the input for MIP_habitat.py
#-----------------------------------------------------------------------
import nlmpy
import numpy as np
import random
from scipy import stats

# This is how we name the cell
# Example: for 5x5 grid
#                | 1| 2| 3| 4| 5|
#                | 6| 7| 8| 9|10|
#                |11|12|13|14|15|
#                |16|17|18|19|20|
#                |21|22|23|24|25|

n=10                # grid size is nxn
nt=n*n              # total number of cells in grid

# -----------------------------------------------------------------------
# Please refer Figure 5.2 for the assumed distribution of
# initial cell fuel age

# Initial cell fuel ages which are 0,1 and 2 years
xk = np.arange(0,3)
pk = (36.0/105.0,35.0/105.0,34.0/105.0)
custmY = stats.rv_discrete(name='custmY', values=(xk, pk))

# Initial cell fuel ages which are from 3 up to 7 years
xnM = np.arange(3,8)
pnM = (33.0/155.0,32.0/155.0,31.0/155.0,30.0/155.0,29.0/155.0)
custmnM = stats.rv_discrete(name='custmnM', values=(xnM, pnM))

# Initial cell fuel ages which are from 8 up to 11 years
xkM = np.arange(8,12)
pkM = (28.0/106.0,27.0/106.0,26.0/106.0,25.0/106.0)
custmM = stats.rv_discrete(name='custmM', values=(xkM, pkM))

# Initial cell fuel ages are from 12 up to 14 years
xr = np.arange(12,15)
pr = (24.0/69.0,23.0/69.0,22.0/69.0)
custmR = stats.rv_discrete(name='custmR', values=(xr, pr))

# Initial cell fuel ages which are from 15 up to 16 years
xkO = np.arange(15,17)
pkO = (21.0/41.0,20.0/41.0)
custmO = stats.rv_discrete(name='custmO', values=(xkO, pkO))

proportion=[0.221, 0.326, 0.223, 0.145, 0.086]

# Random NEUTRAL LANDSCAPE MODELS (NLM),
# please see Etherington et. al (2015) for details
FigA = nlmpy.random(n, n)

# Classified Random NLM
Landscape1 = nlmpy.classifyArray(FigA, [proportion])

Landscape1_age=[[] for i in range(n)]
for i in range(len(Landscape1)):
    for j in range(len(Landscape1[i])):
```

```
        if Landscape1[i][j] in range(1):
            Landscape1_age[i].append(int(custmY.rvs()))
        if Landscape1[i][j] in range(1,2):
            Landscape1_age[i].append(int(custmnM.rvs()))
        if Landscape1[i][j] in range(2,3):
            Landscape1_age[i].append(int(custmM.rvs()))
        if Landscape1[i][j] in range(3,4):
            Landscape1_age[i].append(int(custmR.rvs()))
        if Landscape1[i][j] in range(4,5):
            Landscape1_age[i].append(int(custmO.rvs()))

age=np.array(Landscape1_age)

# Cell fuel age
flatten_Landscape1_age=sum(Landscape1_age,[])
aInitial={}
for i in range(1,nt+1):
    aInitial[i]=flatten_Landscape1_age[i-1]


# Identify the mature cell in the landscape
# We assume that the mature age threshold is 8 years
matureCell=[]
for i in aInitial.keys():
    if aInitial[i] >= 8:
        matureCell.append(i)
    else:
        continue

# ----------------------------------------------------------------------
# We need to count the initial habitat connectivity in the landscape
# First, collect the neighbour indices of cell i in h_adjacent[i]
h_adjacent={}
for i in range(1,nt+1):
        h_adjacent[i]=i

# Corner cells  in nxn rectangular grid
h_adjacent[1]=[2,n+1]                       #Top left corner

h_adjacent[n]=[n-1,2*n]                     #Top right corner
h_adjacent[(n*(n-1)+1)]=[(n*(n-2)+1), (n*(n-1)+2)]      #bottom left
h_adjacent[nt]=[nt-1, (n-1)*n]                          #bottom right
# Left hand boundary column
for i in range(n+1,(n*(n-1)+1),n):
  h_adjacent[i]=[(i-n), (i+n), i+1]

# Right hand boundary column
for i in range(2*n,(n)*n,n):
  h_adjacent[i]=[i-n, i+n,i-1]

# Top row
for i in range(2,(n)):
    h_adjacent[i]=[i-1,i+1,i+n]

# Bottom row
for i in range((n-1)*n+2,(n*n)):
  h_adjacent[i]=[i-1, i+1,i-n]

# Middle
for j in range(n,(n-1)*n,n):
    for i in range(j+2,j+n):
        h_adjacent[i]=[i-1,i+1,i+n,i-n]

```

```
# --- end of determining neighbourhoods for cell i---


# Here, we count the connection only once
# For example: Connections of cells i and j is considered
# the same as the connections of cell j and i
# The connection is is (i,j) where i < j
h_one_sided={}
for i in range(1,nt+1):
        h_one_sided[i]=[]

for i in h_adjacent:
   for j in h_adjacent[i]:
       if j>i:
           h_one_sided[i].append(j)
       else:
           continue

# Finally, count the initial habitat connectivity in the landscape
Habitat_connection_one_sided=[]
for i in h_one_sided.keys():
    if i in matureCell:
        for j in h_one_sided[i]:
            if j in matureCell:
                Habitat_connection_one_sided.append((i,j))


G_t_initially=len(Habitat_connection_one_sided)

# ---------------------------------------------------------------------
# Plot the initial landscape

import matplotlib.pyplot as plt
import matplotlib as mpl

# Make colour map
cmapClas = mpl.colors.ListedColormap([ '#bebada', '#8dd3c7', \
'#ffffb3','#fb8072'])
bounds=[0,1,2,3,4]
norm = mpl.colors.BoundaryNorm(bounds, cmapClas.N)

# Create figure
plt.subplot2grid((6,6), (0,0),rowspan=5,colspan=4)
mpl.rc('axes', linewidth=0.5) # set all axes line widths

plt.xticks(np.arange(0))
plt.yticks(np.arange(0))
plt.imshow(Landscape1, interpolation='none', cmap=cmapClas, norm=norm)

# Display the age of each cell
for y in range(Landscape1.shape[0]):
    for x in range(Landscape1.shape[1]):
        plt.text(x + 0.05, y + 0.05, '%s' % age[y, x],
                horizontalalignment='center',
                verticalalignment='center',fontsize=12
                )

# Plot legend
plt.subplot2grid((6,6), (5,0),colspan=3)
x = np.array([np.array(np.repeat(range(4), 5))])
plt.imshow(x, interpolation='none', aspect=1, cmap=cmapClas, norm=norm)
plt.yticks(np.arange(0))
plt.xticks([2,7,12,17], ["<minTFI","non-mature","mature",\
```

```
"mature&high fuel"], fontsize=8)
plt.tick_params(direction='out', length=4, width=0.5, top='off')
plt.title("Cell's fuel age (year)", fontsize=8)

plt.savefig("Initial_cell_fuel_age") #save as .png file
# ----------------------------------------------------------------------
# Please see the output file and copy this lists as the input for MEE_MIP.py
print "aInitial=",aInitial
print "G_t_initially=",G_t_initially
#-----------------------------------------------------------------
```

```
# ----------------------------------------------------------------------------
# MIP_habitat.py
# ----------------------------------------------------------------------------

# This script is used for running the computational experiments in Chapter 5 by
# by writing the LP file and solve it



# Here we use the initial cell fuel age data from Figure 5.5
# The fuel treatment schedule will be the same as that of presented in Figure 5.5
# if we run this code for 13 year planning horizon

aInitial= {1: 3, 2: 13, 3: 12, 4: 7, 5: 2, 6: 13, 7: 10, 8: 2, 9: 16, 10: 2, \
11: 6, 12: 8, 13: 2, 14: 1, 15: 5, 16: 16, 17: 1, 18: 4, 19: 0, 20: 4, \
21: 5, 22: 2, 23: 11, 24: 3, 25: 8, 26: 10, 27: 8, 28: 5, 29: 8, 30: 4, \
31: 6, 32: 11, 33: 4, 34: 10, 35: 13, 36: 16, 37: 14, 38: 4, 39: 14, 40: 10, \
41: 9, 42: 4, 43: 4, 44: 16, 45: 10, 46: 0, 47: 3, 48: 9, 49: 11, 50: 14,\
51: 4, 52: 1, 53: 16, 54: 2, 55: 14, 56: 11, 57: 8, 58: 4, 59: 15, 60: 6, \
61: 4, 62: 1, 63: 2, 64: 4, 65: 1, 66: 4, 67: 11, 68: 5, 69: 0, 70: 0, \
71: 16, 72: 15, 73: 15, 74: 0, 75: 14, 76: 7, 77: 7, 78: 8, 79: 3, 80: 7, \
81: 2, 82: 12, 83: 13, 84: 0, 85: 11, 86: 6, 87: 2, 88: 2, 89: 11, 90: 7, \
91: 7, 92: 12, 93: 13, 94: 14, 95: 10, 96: 7, 97: 6, 98: 12, 99: 2, 100: 11}

G_t_initially=39    # initial number of habitat connectivity of the landscape

# For another initial cell fuel age,
# We need to run Landscape.py and copy the output (aInitial and G_t_initially)
# here


from pulp import *
import sys
import os
import cplex
import time
import nlmpy
import numpy as np
import random

# Example: for 5x5 grid, here is the cell's name
#                | 1| 2| 3| 4| 5|
#                | 6| 7| 8| 9|10|
#                |11|12|13|14|15|
#                |16|17|18|19|20|
#                |21|22|23|24|25|

# ----------------------------------------------------------------------------
setting=1       # (i) Gt is set to initial number of habitat connectivity \
            # of the landscape.
            # (ii) Neighbouring habitat cell requirement \
            # for treatment:yes (Constraint 12 is applied)

            # NOTE: Please see the section 5.2 for details:
            # For setting=2, G_t_initially=39; Constraint_12 is not applied
            # For setting=3, G_t_initially=0;  Constraint_12 is applied
            # For setting=4, G_t_initially=0;  Constraint_12 is not applied

n=10                # grid size is nxn
nt=n*n          # total number of cells in grid
totalt=11       # planning horizon (plus 1) e.g. if we plan for 10 years ahead, \
                # then totalt=11
```

```
rho=10          # treatment level (in percent), 10 means 10% treatment level
M=30            # Big M
MinTFI_i=2      # minimum Tolerable Fire Interval of cell i
m_i=8           # mature age threshold for cell i
d_i=12          # high fuel load age threshold for cell i
MaxTFI_i=16     # maximum Tolerable Fire Interval of cell i
max_age=50      # maximum age
phase1=0        # 1 means we need 1 year for phase 1
G_t_phase1=[]
G_t={}          # desired target of mature cell connectivity in time t
for i in range(1,totalt):
    if i <phase1+1:
        G_t[i]=G_t_phase1[i-1]
    else:
        G_t[i]=G_t_initially
#

# ----------------------------------------------------------------------------
# NOTE: for some landscapes, it is impossible to maintain the initial number of
# habitat over the planning horizon. To deal with this infeasibility,
# we ran the model by assigning a lower value of G_t (for example, half of
# G_t_initially) for the first years in a planning horizon, and setting the \
# higher value (G_t_initially) for the rest of the year within the planning \
# horizon only once it is feasible.
# In this script, suppose that we need two years to run Phase 1, then write:
# phase1=2; G_t_phase1=[19,19]
# ----------------------------------------------------------------------------


# Collect the neighbour indices of cell i in h_adjacent[i]
h_adjacent={}
for i in range(1,nt+1):
        h_adjacent[i]=i

# Corner cells  in nxn rectangular grid
h_adjacent[1]=[2,n+1]                   # Top left corner
h_adjacent[n]=[n-1,2*n]                 # Top right corner
h_adjacent[(n*(n-1)+1)]=[(n*(n-2)+1), (n*(n-1)+2)]      # bottom left
h_adjacent[nt]=[nt-1, (n-1)*n]                          # bottom right

# Left hand boundary column
for i in range(n+1,(n*(n-1)+1),n):
  h_adjacent[i]=[(i-n), (i+n), i+1]

# Right hand boundary column
for i in range(2*n,(n)*n,n):
  h_adjacent[i]=[i-n, i+n,i-1]

# Top row
for i in range(2,(n)):
    h_adjacent[i]=[i-1,i+1,i+n]

# Bottom row
for i in range((n-1)*n+2,(n*n)):
  h_adjacent[i]=[i-1, i+1,i-n]

# Middle
for j in range(n,(n-1)*n,n):
    for i in range(j+2,j+n):
        h_adjacent[i]=[i-1,i+1,i+n,i-n]

# ------------- end of determining neighbourhoods for cell i ----------------
```

```

# Here, we count the connection only once
h_one_sided={}
for i in range(1,nt+1):
        h_one_sided[i]=[]

for i in h_adjacent:
   for j in h_adjacent[i]:
       if j>i:
           h_one_sided[i].append(j)
       else:
           continue

# ---------------------------------------------------------------------------

# Define the decision variables. Please see Chapter 5 for details

# Decision variable A_it
# The age of vegetation in cell i, at time t
A_it={}
for i in range(1,nt+1):
    for t in range(totalt):
        A_it[i,t]=LpVariable("%1s%1s%1s%1s%1d"%("A",",",i,",",t),0,max_age,\
        LpInteger)


# Decision variable x_it
# Indicates a cell to burn at time t
x_it={}
for i in range(1,nt+1):
    for t in range(1,totalt):
        x_it[i,t]=LpVariable("%1s%1s%1s%1s%1d"%("x",",",i,",",t),0,1,LpInteger)

# Decision variable Mature_it
Mature_it={}
for i in range(1,nt+1):
    for t in range(1,totalt):
        Mature_it[i,t]=LpVariable("%1s%1s%1s%1s%1d"%("Mature",",",i,",",t),\
        0,1,LpInteger)

# Decision variable HabitatConn_ijt
HabitatConn_ijt={}
for i in h_one_sided.keys():
    for j in h_one_sided[i]:
        for t in range(1,totalt):
            HabitatConn_ijt[i,j,t]=LpVariable("%1s%1s%1s%1s%1s%1s%1s"%\
            ("HabitatConn",",",i,",",j,",",t),0,1,LpInteger)

# Decision variable High_it
High_it={}
for i in range(1,nt+1):
    for t in range(1,totalt):
        High_it[i,t]=LpVariable("%1s%1s%1s%1s%1d"%("High",",",i,",",t),0,1,\
        LpInteger)


# Decision variable HighConn_ijt
# Indicates whether two high fuel load cells are adjacent
HighConn_ijt={}
for i in h_one_sided.keys():
    for j in h_one_sided[i]:
        for t in range(1,totalt):
```

```
            HighConn_ijt[i,j,t]=LpVariable("%1s%1s%1s%1s%1s%1s%1s"%\
            ("HighConn",",",i,",",j,",",t),0,1,LpInteger)

# Decision variable Old_it
Old_it={}
for i in range(1,nt+1):
    for t in range(totalt-1):
        Old_it[i,t]=LpVariable("%1s%1s%1s%1s%1d"%("Old",",",i,",",t),0,1,\
        LpInteger)

# ---------------------------------------------------------------------------
# The constraints. Please see Section 5.2 for details
constraint_2={}
constraint_3={}
constraint_4={}
constraint_5={}
constraint_6={}
constraint_7={}
constraint_8={}
constraint_9={}
constraint_10={}
constraint_11={}
constraint_12={}
constraint_13={}
constraint_14={}
constraint_15={}
constraint_16={}
constraint_17={}
constraint_18={}
constraint_19={}


for t in range(1,totalt):
    for i in range(1,nt+1):
        constraint_6[i,t]=LpConstraint(lpSum(A_it[i,t]-A_it[i,t-1]),\
        sense=-1,rhs=1)
        constraint_5[i,t]=LpConstraint(lpSum(A_it[i,t]+M*x_it[i,t]),\
        sense=-1,rhs=M)
        constraint_4[i,t]=LpConstraint(lpSum(A_it[i,t]-A_it[i,t-1]+\
        M*x_it[i,t]),sense=1,rhs=1)
        constraint_16[i,t]=LpConstraint(lpSum(A_it[i,t-1]-\
        M*Old_it[i,t-1]),sense=-1,rhs=MaxTFI_i-1)
        constraint_17[i,t]=LpConstraint(lpSum(A_it[i,t-1]-\
        MaxTFI_i*Old_it[i,t-1]),sense=1,rhs=0)
        constraint_19[i,t]=LpConstraint(lpSum(A_it[i,t-1]-\
        MinTFI_i*x_it[i,t]),sense=1,rhs=0)
        constraint_10[i,t]=LpConstraint(lpSum(A_it[i,t]-\
        M*Mature_it[i,t]),sense=-1,rhs=m_i-1)
        constraint_11[i,t]=LpConstraint(lpSum(A_it[i,t]-\
        m_i*Mature_it[i,t]),sense=1,rhs=0)
        constraint_7[i,t]=LpConstraint(lpSum(A_it[i,t]-\
        M*High_it[i,t]),sense=-1,rhs=d_i-1)
        constraint_8[i,t]=LpConstraint(lpSum(A_it[i,t]-\
        d_i*High_it[i,t]),sense=1,rhs=0)
        constraint_2[t]=LpConstraint(lpSum([x_it[i,t] for \
        i in range(1,nt+1)]),sense=-1,rhs=rho*n*n*0.01)
        constraint_15[t]=LpConstraint(lpSum([HabitatConn_ijt[i,j,t]  \
        for i in range(1,nt+1) for j in h_one_sided[i]]),sense=1,rhs=G_t[t])

for k in range(1,nt+1):
    constraint_3[k]=LpConstraint(lpSum(A_it[k,0]),sense=0,rhs=aInitial[k])

```

```
for t in range(1,totalt):
    for i in range(1,nt+1):
        for j in h_adjacent[i]:
            constraint_12[i,t]=LpConstraint(lpSum([Mature_it[j,t] for j \
            in h_adjacent[i]]-x_it[i,t]),sense=1,rhs=0)
            constraint_18[i,t]=LpConstraint(lpSum(4*Old_it[i,t-1]+\
            [Mature_it[j,t] for j in h_adjacent[i]]-4*x_it[i,t]),\
            sense=-1,rhs=4)

for t in range(1,totalt):
    for i in range(1,nt+1):
        for j in h_one_sided[i]:
            constraint_13[i,j,t]=LpConstraint(lpSum(Mature_it[i,t] + \
            Mature_it[j,t]-HabitatConn_ijt[i,j,t]),sense=-1,rhs=1)
            constraint_14[i,j,t]=LpConstraint(lpSum(Mature_it[i,t] + \
            Mature_it[j,t]-2*HabitatConn_ijt[i,j,t]),sense=1,rhs=0)
            constraint_9[i,j,t]=LpConstraint(lpSum(High_it[i,t] + \
            High_it[j,t]-HighConn_ijt[i,j,t]),sense=-1,rhs=1)
# ---------------------------------------------------------------------------
# Create the 'prob_burnplan' variable to contain the problem data
prob_burnplan = LpProblem("%10s"%("The Fuel Treatment Planning maintaining \
Habitat Availability and Connectivity for Species Conservation "), LpMinimize)
# The objective function is added to 'prob_burnplan'
prob_burnplan += lpSum([HighConn_ijt[i,j,t] for t in range(1,totalt) for i \
in range(1,nt+1)    for j in h_one_sided[i]]),"Tot high fuel load cell connectivity"

# The constraints are added to 'prob_burnplan'
for k in constraint_2.keys():
        prob_burnplan += constraint_2[k]
for k in constraint_3.keys():
        prob_burnplan += constraint_3[k]
for k in constraint_4.keys():
        prob_burnplan += constraint_4[k]
for k in constraint_5.keys():
        prob_burnplan += constraint_5[k]
for k in constraint_6.keys():
        prob_burnplan += constraint_6[k]
for k in constraint_7.keys():
        prob_burnplan += constraint_7[k]
for k in constraint_8.keys():
        prob_burnplan += constraint_8[k]
for k in constraint_9.keys():
        prob_burnplan += constraint_9[k]
for k in constraint_10.keys():
        prob_burnplan += constraint_10[k]
for k in constraint_11.keys():
        prob_burnplan += constraint_11[k]
for k in constraint_12.keys():
        prob_burnplan += constraint_12[k]
for k in constraint_13.keys():
        prob_burnplan += constraint_13[k]
for k in constraint_14.keys():
        prob_burnplan += constraint_14[k]
for k in constraint_15.keys():
        prob_burnplan += constraint_15[k]
for k in constraint_16.keys():
        prob_burnplan += constraint_16[k]
for k in constraint_17.keys():
        prob_burnplan += constraint_17[k]
for k in constraint_18.keys():
        prob_burnplan += constraint_18[k]
for k in constraint_19.keys():
```

```
        prob_burnplan += constraint_19[k]

# File name (lp file)
file="lp_file1_%dgrids_%dpercent_%dyear_setting_%s.lp"%(n*n,rho,totalt-1,setting)

# Write the lp file
prob_burnplan.writeLP(file)
# ---------------------------------------------------------------------------

c = cplex.Cplex(file)
c.objective.set_sense(c.objective.sense.minimize)
c.parameters.timelimit.set(3600) # Limit the solution time (in seconds)
time_start = time.clock()
# Solve the lp file
c.solve()
time_elapsed = (time.clock() - time_start)
print "time_elapsed=",time_elapsed
print "Status:",c.solution.get_status_string()
print "Total high fuel load cell connectivity= ", \
c.solution.get_objective_value()

# Print the solution
for i, v in enumerate(c.solution.get_values()):
    if v >= 0.001:

        print "%-20s%-2s%-20s"%(str(c.variables.get_names(i)),"=",str(v))
# ---------------------------------------------------------------------------
VarName=c.variables.get_names()
SolValue=c.solution.get_values()

# Treatment schedule
burn_in_t=[[] for i in range(1,totalt)]
for j,i in enumerate(VarName):
    for k in range(1,nt+1):
        for t in range(1,totalt):
            if str(x_it[k,t]) == i:
                if SolValue[j]>0.001:

                    burn_in_t[t-1].append(k)
print " "
print "Fuel treatment schedule"
for i in range(len(burn_in_t)):
    print "Treated cells in year",i+1,":",burn_in_t[i]
# ---------------------------------------------------------------------------
```